Correspondence: Why Kant Can Konnect Mathematics to Experience

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Introduction

One of the most divisive aspects of Kant's *Critique of Pure Reason* comes about due to his views on mathematics. He believed both arithmetical and geometrical judgments were undeniably paragon examples of synthetic *a priori* judgments. Rather than falling into Hume's skepticism[1], Kant wishes to ground this faculty for judgment entirely *a priori*, so as to avoid mixing an empirical or probabilistic element into what should be pure and objective judgment. Following this task, this thesis will ask: "Why is there a necessary correlation between mathematics and objects of experience?" Furthermore, why do *a priori* mathematics determine our human capacity to communicate about the objective world of experience?

To demonstrate the necessity of this question, I will allow Kant to speak for himself:

> Since the propositions of geometry [and arithmetic] are cognized synthetically *a priori* and with apodictic certainty, I ask: Whence do you take such proposition and on what does our understanding rely in attaining to such absolutely necessary and universally valid truths? There is no other way than through concepts or through intuitions. (A47)

Kant asks "by what means are concepts and intuitions are related to the propositions (judgments) of geometry?" Mathematics determine laws through

[1] "Geometry, or the technique by which we fix the proportions of figures, never achieves perfect precision and exactness (though its results are much more general and exact than the loose judgments of the senses and imagination). Its first principles are drawn from the general appearance of the objects" (Hume, *Treatise*, 1.iii.1). If Hume is correct, this would necessitate including a contingent element into what should be apodictic judgments, which Kant wishes to avoid.

"absolutely necessary and universally valid truths." The question is, then, how are we, as human beings, able to affirm without doubt that Geometric representation of triangles determine the properties of physical triangular objects, and on *a priori* grounds?

By investigating the nature of these problems, I hope to assist Kant in demonstrating the necessary correspondence of the world of experience to the world of pure mathematics. In this pursuit I will explicate the role of intuitions and concepts in this determination, the role of the Transcendental Aesthetic in this theory, and finally culminate in the Transcendental Analytic and Deduction.

Transcendental Aesthetic

In the *Critique of Pure Reason*, Kant defines both space and time as components, or grounds of possibility, of the Transcendental Aesthetic. They are the first *a priori* synthetic intuitions that humans are imbued with, and thus constitute the pure intuitions. Directly stemming from this discovery, Kant marks mathematics as being derived from these pure intuitions, and thus must necessarily partake in being *a priori*. Thus, if both arithmetic and geometry emerge from our intuitions, and are themselves equally *a priori*, then how can they directly relate to the state of reality? For as stated in the introduction, "mathematics gives us a splendid example of how far we can go with *a priori* cognition independently of experience" (Kant, B8-9). Mathematics here means our ability to make judgments about "how much" of things, similar to the Latin *quantus*. This definition of mathematics can be summed up as our ability to judge and measure any object. Since Kant affirms mathematical principles as universally valid due to their origin and *a priori* status, this essay will investigate the nature of both arithmetic and geometry and their relationship to existing things.

To begin is to lay down what must be accomplished. There must first be understanding of the Transcendental Aesthetic and its components, followed by an investigation into the Transcendental Analytic and the Transcendental Deduction. Elucidation of what has been gained for mathematics will follow each section. Finally, the importance of math and its correspondence as a mode of synthetic judgement to objects of experience will become apparent by the end of the paper.

To understand both arithmetic and geometry in the Kantian system, the reader must first understand what components have gone into making the mathematicals. The pieces of the Aesthetic come into view to oneself from **abstraction**; "If I separate from the representation of a body that which the understanding thinks about it….something from this empirical intuition is still left for me, namely extension and form" (A21). The first of these components, and the pure intuition that relates to extension, is Space. Space is characterized by its nature as the outer sense, as "we represent to ourselves objects as outside us, and all as in space" (B37). Space is a necessary representation that creates the "condition of the possibility of appearances." (B39) Not only is this representation necessary for cognition of external objects, but also grounds "all geometric principles... [which] are never derived from general concepts of line and triangle, but rather are derived from intuition and indeed derived *a priori* with apodictic certainty" (B40). Space, as the condition of geometric principles, is also what gives them their universality in their ability to be used mathematically in the measurement of objects. Without the universality grounded on pure intuition, then the application of geometric principles would start to require individual proof for every use by being grounded empirically, and thus would cease to be *a priori* principles at all. Thus, "our explanation alone makes the possibility of geometry as a synthetic *a priori* cognition comprehensible" by establishing it as sired from the pure intuition of space.

In a similar way Time, the other pure intuition, represents the inner sense. Whereas space grounds external objects, and posits them as outside ourselves through the device of spaces, time grounds "simultaneity or succession" (B46). Without this capacity being given *a priori*, then one cannot represent several things existing sequentially or simultaneously. Thus, "Time is a necessary representation that grounds all intuitions...In it alone is all actuality of appearance possible" (A31). Whereas Space creates the capacity to receive appearances by cognizing them externally, Time is what actualizes the existence of appearance. This delineation is reinforced by Kant himself in his conclusions, since

> [s]pace, as the pure form of all outer intuitions, is limited as an *a priori* condition merely to outer intuitions. But since, on the contrary, all representations, whether or not they have other things as their object, nevertheless as determinations of the mind themselves belong to the inner state, while this inner state belongs under the formal condition of inner intuition, and thus of time, so time is an *a priori* condition of all appearances in general...and thereby also the mediate condition of outer appearances. (A34)

Only by making both of the intuitions necessary aspects of the human sensibility towards appearances does Kant prove and demonstrate the empirical reality of space apodictically, through the reality of the objects of the inner sense being immediately clear. Space's (objective) reality comes to be "in regard to everything that can come before us externally as an object" while at the same time only existing as a "transcendental ideality, i.e., that it is nothing as soon as we leave out the conditions of the possibility of all experience, and take it as something that grounds the things in themselves" (B44). When space is considered "in regard to things when they are considered in themselves through reason" it is only ideality, but through human sensibility it has "empirical reality" (B44). In a similar way, "[i]f we abstract from our way of internally intuiting ourselves and by means of this intuition...and thus take objects as they may be in themselves, then time is nothing" (B51). It only retains objective validity "in regard to appearances, because these are already things that we take as objects of our senses...Time is therefore merely a subjective condition of our (human) intuition . . . Nonetheless it is necessarily objective in regard to all appearances" (B52). Due to this objective reality when dealing with appearances in general, and thus serving as the grounding of all intuitions of sensibility, "time and space are accordingly two sources of cognition, from which different synthetic cognitions can be drawn *a priori*, of which especially pure mathematics in regard to the cognition of space and its relations provide a splendid example" (B56).

Finally, in the Aesthetic Elucidation, the reader reaches the problem of correspondence.

> For if it were to be supposed that space and time are in themselves objective and conditions of the possibility of things in themselves, then it would be shown, first, that there is a large number of *a priori* apodictic and synthetic propositions about both, but especially about space...Since the propositions of geometry are cognized synthetically *a priori* and with apodictic certainty, I ask: Whence do you take such proposition and on what does our understanding rely in attaining to such absolutely necessary and universally valid truths? There is no other way than through concepts or through intuitions. (A47)

After proving the necessity of intuitions to arrive at ampliative synthetic *a priori* judgements, Kant outlines the method:

> You must therefore give your object *a priori* in intuition, and ground your synthetic proposition on this. If there did not lie in you a faculty for intuiting *a priori*; if this subjective condition regarding form were not at the same time the universal *a priori* condition under which alone the object of this (outer) intuition is itself possible; if the object (the triangle) were something in itself without relation to your subject: then how could you say that what necessarily lies in your subjective conditions for constructing a triangle must also necessarily pertain to the triangle in itself. (B66; emphasis added)

Thus, the reader has finally arrived at the difficulty of correspondence. Furthermore, there is also understanding of the difficulty of the problem; If mathematics lose their relationship to pure intuitions, or if the faculty of judgment is unable to reason on *a priori* synthetics, then mathematics is dead in the water and has always had an illusory existence. This is the view of Hume who, lacking an objective grounding for mathematics based on Space and Time, instead is necessitated to attribute its origin to empiricism and thus damns Geometry to universal imprecision.

> When geometry decides anything concerning the proportions of quantity, we shouldn't expect the utmost precision and exactness—none of its proofs yield that. Geometry takes the dimensions and proportions of figures accurately—but roughly, with some give and take. Its errors are never considerable, and it wouldn't it err at all if it didn't aim at such an absolute perfection. (Hume, *Treatise,* 1. ii. 3)

To finish the Transcendental Aesthetic, Kant leaves the reader with a question which runs through the course of his argument; " How are synthetic *a priori* propositions possible?" (B73) The intuitions of time and space form the first piece of the puzzle, as

> if we want to go beyond the given concept in an *a priori* judgment, we encounter that which is to be discovered *a priori* synthetically connected with it, not in the concept but in the intuition that corresponds to it; but on

> this ground such a judgment never extends beyond the objects of the senses and can hold only for objects of possible experience. (B73)

Thus, under this framing, the pure intuition of mathematics can only correspond to objects of possible experience, or appearances. How will this affect the pursuit of correspondence?

Transcendental Logic

The next piece of the puzzle is the Transcendental Logic. For

> if we will call the receptivity of our mind to receive representations insofar as it is affected in some way sensibility, then on the contrary the faculty for bringing forth representation itself, or the spontaneity of cognition, is the understanding. (B75)

Whereas the Aesthetic was concerned with sensibility and thus the pure intuitions, the Logic is instead concerned with the faculty of understanding, and thus thinking. For

> logic in turn can be undertaken with two different aims, either the logic of the general or of the particular use of the understanding. The former contains the absolutely necessary rules of thinking, without which no use of the understanding takes place. (B76)

Just as Time and Space were the forms of inner and outer sense, General logic is the form of thinking and of the understanding. Kant elucidates their necessary relationship: "Thoughts without content are empty, intuitions without concepts are blind" (B76). Thus, the ability to synthesize new concepts through the use of the understanding is required to make use of any data from sensibility.

Though general logic relates to the form of understanding, it can be further subdivided into pure, or transcendental logic. Whereas general logic is the foundation of thought itself, transcendental logic relates to objects as given through the manifold of sensibility, or the Transcendental Aesthetic. The definition of this subsection of general logic is:

> such a science, which would determine the origin, the domain, and the objective validity of such cognitions, would have to be called transcendental logic, since it has to do merely with the laws of the understanding and reason, but solely insofar as they are related to objects *a priori.* (B82)

This definition is further refined:

> Not every *a priori* cognition must be called transcendental , but only that by means of which we cognize that and how certain representations (intuitions or concepts) are applied entirely *a priori* or are possible. (B81)

Thus,

> neither space nor any geometrical determination of it *a priori* is a transcendental representation, but only the cognition that these representations are not of empirical origin at all and the possibility that they can nevertheless be related *a priori* to objects of experience can be called transcendental. (B81)

The question of correspondence is, at least in part, about the nature of the transcendental in mathematics. If there is a true correspondence, it can only come about through the existence of Transcendental Logic, which explicitly relates the manifold of pure intuition through Space and Time to the representations of the understanding, and thus allows for judgment on objects of experience.

To draw valid inferences on the topic of correspondence, it must first be questioned: What is Truth? Kant defines it nominally as “the agreement of cognition with its object” (A58). However, this does not penetrate deeply enough, as “one demands to know what is the general and certain criterion of the truth of any cognition" (A58). To answer this absurd request, "one must therefore say that no general sign of the truth of the matter of cognition can be demanded, because it is self-contradictory"[2] (A59). However, the question of truth must still be further understood, since "a logic, so far as it expounds the general and necessary rules of understanding, must present criteria of truth in these very rules" (B84). To further refine this definition, a "merely logical criterion of truth, namely the agreement of a cognition with the general and formal laws of understanding and reason is there certainly the *conditio sine qua non*, and thus the negative condition of all truth; further, however, logic cannot go" (A60). This negative condition of truth, which is constrained by both formal and general laws of the understanding, will manifest itself as the Categories in the Transcendental Analytic, and thus become a keystone in Kant's argument both generally and for the correspondence of mathematics.

[2] The contradiction is best seen as an infinite regress, as any answer to the question "what is the general sign of the truth" can be questioned again, with the question "what is the general sign of the truth of the general sign of the truth" repeated *ad infinitum*. Thus, Kant concludes that no general sign can be given to avoid this logical pitfall.

Transcendental Analytic

After distinguishing "the science of the rules of sensibility in general, i.e., the transcendental aesthetic, from the science of the rules of understanding in general, i.e., logic" (B76), and Pure Logic from Applied General Logic, Kant begins to dive into his Transcendental Analytic. The purpose of this exercise is crucial to the argument, since *a priori* synthetic judgments must necessarily fall under the purview of "this Analytic [which] is the analysis of the entirety of our *a priori* cognition into the elements of the pure cognition of the understanding" (B89). Kant thus seeks to fully catalogue what faculties are already present using his four points:

> 1. That the concepts be pure and not empirical concepts. 2. That they belong not to intuition and to sensibility, but rather to thinking and understanding. 3. That they be elementary concepts, and clearly distinguished from those which are derived or composed from them. 4. That the table of them be complete and that they entirely exhaust the entire field of pure understanding. (B90)

Furthermore, this catalogue would make clear "their connection in a system . . . [the system] is therefore a unity...which is not to be supplemented by any external additions. Hence, the sum total of its condition will constitute a system that is to be grasped and determined under one idea" (B90). The name of this system would be the categories, pure concepts of the understand given *a priori*.

The Logical Functions of the Understanding in Judgments

Quantity of Judgments	Quality of Judgments	Relation of Judgments	Modality of Judgments
Universal	Affirmative	Categorical	Problematic
Particular	negative	Hypothetical	Assertoric
Singular	Infinite	Disjunctive	Apodictic

Rather than beginning with the categories, it would benefit the reader to trace the argument from the beginning of the Analytic and see how it stems from judgment. Rather than starting out with all possible faculties of understanding, the categories, the author instead exhausts the ledger of possible types of judgments, or the table of Logical Functions. This list of judgments will be correlated with the categories, as “if we abstract from all content of a judgment in general, and attend only to the mere form of the understanding in it, we find that the function of thinking in that can be brought under four titles” (B95). In a similar way, “there arise exactly as many pure concepts of the understanding...as there were logical functions of all possible judgments in the previous table . . .Following Aristotle we will call these concepts categories” (B80). While the two tables are isomorphic, Kant seems to indicate in his personal notes before the categories that their functions differ significantly; The table of judgments “are only forms for the relation of concepts in thinking,” while “Categories are concepts, through which certain intuitions are determined in regard to synthetic unity...as contained under these functions” (B80). Thus both tables are related, and this unity under the headings of

Quantity, Quality, Relation, and Modality will be demonstrated later in this essay. However, since the categories were "systematically generated from a common principle, namely the faculty for judging," (B107) it is only with the elucidation of the table of judgments that Kant is able to progress to the categories.

> Concepts are therefore grounded on the spontaneity of thinking, as sensible intuitions are grounded on the receptivity of impressions. Now the understanding can make no other use of these concepts than that of judging by means of them. Since no representation pertains to the object immediately except intuition alone, a concept is thus never immediately related to an object, but is always related to some other representation of it...Judgment is therefore the mediate cognition of an object. (B94, emphasis added)

Furthermore, "we can...trace all actions of the understanding back to judgments, so that the understanding in general can be represented as a faculty for judging" (B94). The common root of this process, regardless of what is being judged, is a key aspect of Kant's General Logic, since "the functions of the understanding can therefore all be found together if one can exhaustively exhibit the functions of unity in judgments" (B95). Necessarily, the categories must complete the entire field of pure understanding, and thus be a catalogue of all possible modes of judgment. As a type of judgment mathematics must also display this unity especially if is to lay claim to the strongest modality possible, apodictic certainty, which is "determined through these laws of the understanding itself, and as thus assert[ed] *a priori*" (B102).

Continuing on from judgments, Kant examines General Logic and Transcendental Logic. General Logic, as the "abstract[ion] from all content of cognition," involves only formal relations, and can be used indifferently on both *a priori* and empirical content without distinction. In contrast, Transcendental Logic "has a manifold of sensibility that lies before it *a priori*, which the Transcendental Aesthetic has offered to it, in order to provide the pure concepts of the understanding with matter" (B103). The Transcendental Aesthetic "contain[s] a manifold of pure *a priori* intuition, but belong nevertheless among the conditions of the receptivity of our mind" (B103). With both the manifold and receptivity "under which alone it can receive representation of objects, [so that] they must always also affect the concepts of these objects" (B103), the Transcendental Aesthetic requires a special process to combine together its *a priori* intuitions with its capacity to contain appearances. This operation "of our thought requires that this manifold first be gone through, taken up, and combined in a certain way in order for a cognition to be made out of it. I call this action synthesis." Synthesis is thus defined as "the action of putting different representations together with each other and comprehending their manifoldness in one cognition" (B103). Furthermore, "such a synthesis is pure if the manifold is given not empirically but *a priori* (as is that in space and time)" (B103). The important distinction between General and Transcendental Logic is this: Through the use of the Trans. Logic pure concepts of the understanding, or the categories, can be applied to the manifold of sensibility.

> The same understanding, therefore, and indeed by means of the very same actions through which it brings the logical form of a judgment into concepts by means of the analytical unity also brings a transcendental content into its representation by means of the synthetic unity of the manifold in intuition in general, on account of which they are called pure concepts of the understanding that pertain to objects *a priori*; this can never be accomplished by general logic. (B105)

Furthermore, this explicitly connects the logical functions table and the categories, as "the same function that gives unity to the different representation in a judgment also gives unity to the mere synthesis of different representations in an intuition, which, expressed generally, is called a pure concept of the understanding" (B105). Transcendental Logic, by explicitly connecting the *a priori* sensibility of the Aesthetic and the pure concept of the understanding generated by "pure synthesis generally represented", (B104) will be a key piece of Kant's argument for correspondence by leading into the "concepts that give this pure synthesis unity, and that consist solely in the representation of this necessary synthetic unity", (B105) which will be discussed in detail in the Transcendental Deduction.

Defining Synthesis and thus syntheticity is paramount to both Kant's general thesis and his interpretation of mathematics, since the capacity of "putting together different representations" (B103) is a necessary requirement for any judgment*, a priori* or empirical, to be made. Synthesis is also necessary to create the "unity" required to mediate the understanding and intuition. Since this synthesis is pure "if the manifold is given not empirically but *a priori*," (B103) Transcendental Logic must necessarily participate in pure synthesis if it wishes to build to the pure concepts of the understanding, since

> pure synthesis, generally represented, yields the pure concept of the understanding. By this synthesis, however, I understand that which rests on a ground of synthetic unity *a priori*; thus our counting (as is especially noticeable in the case of larger numbers) is a synthesis in accordance with concepts. (B104)

Kant needs to utilize this view of synthesis in order to reach both the mathematicals and the categories, and thus to demonstrate that they "are . . . pure concepts of the understanding that pertain to objects *a priori* " (B106). The objects which Kant is relating mathematics to are “objects of intuition in general,” (B106) which conforms with the formal ability of the sensibility, through the Transcendental Aesthetic, to allow for representations of any object. This relation to objects emerges only through the Transcendental Logic, which presumes a synthetic connection between the pure concepts of the understanding and the capacity to receive impressions from sensibility through space and time. Transcendental Logic, as the rule for the use of the understanding, grounds all objects and thus all judgments *a priori* on the Transcendental Aesthetic. Mathematics thus must have a certain connection to objects of experience as a pure concept of the understanding if it is related to them through Transcendental Logic.

Table of Categories

Quantity	Quality	Relation	Modality
Unity	Reality	Inherence and Subsistence	Possibility - Impossibility
Plurality	Negation	Causality and Dependence	Existence - Non Existence
Totality	Limitation	Community	Necessity - Contingency

Now having found the boundaries of the field of judgements in the categories, the author lays down “as many pure concepts of the understanding….as there were logical functions of all possible judgments in the previous table; for the understanding is completely exhausted and its capacity entirely measured by these functions” (B106). Following this, Kant lays down and divides the categories between the mathematical and dynamical. Quantity and Quality are found under this umbrella of mathematical. Whereas counting was a byproduct of the synthetic process, Number itself emerges from Quantity, notably Allness, which "is nothing other than plurality considered as a unity" (B111).

Following the introduction of the categories, Kant has outlined all the possible uses of the understanding in a formal sense. The importance of placing mathematical objects, such as number and line, under the categories of Quantity and Quality conforms with their origin from the Transcendental Aesthetic: The formal capacity to perform mathematical judgments was inherently *a priori* because of their participation in the mathematical categories. To state that in another way, any judgment under the formal categories is *a priori*, and thus mathematical judgments are *a priori* due to their use of the categories of Quantity and Quality. To prove this statement, Kant necessarily resorts to the Transcendental Deduction to "establish the entitlement or legal claim" (B117) of mathematics in its *a priori* use.

Transcendental Deduction

Continuing from his argument that the categories exhaust the field of possible cognitions, Kant turns to the next step of his Critique: "proof...to establish the entitlement, or….the deduction." (B117) At this juncture, mathematics find themselves on trial along with their fellow *a prior*i concepts to justify their "pure use *a priori*." (B118) This deduction would show the entitlement mathematics has to being fully *a priori*, "since proofs from experience are not sufficient for the lawfulness of such a use, and yet one must know how these concepts can be related to objects that they do not derive from any experience." (B118) Furthermore, the deduction must also solve the problem of relation: What right do non-empirical or pure concepts have to determine empirical objects? Kant answers this rhetorical question by noting "I therefore call the explanation of the way in which concepts can relate to objects *a priori* their transcendental deduction, and distinguish this from the empirical deduction." (B118) This transcendental deduction will lead to "the beginning of a deduction of the pure concepts of the understanding" (B145) in relation to the categories uncovered in the Analytic.

Through Time and Space, or the Transcendental Aesthetic, Kant argues that the reader has already received a deduction. By showing that "what is distinctive in their nature is precisely that they are related to objects without having borrowed anything from experience for their representation" then they cannot be the subject of an empirical deduction. Thus, "if a deduction of them is necessary, it must always be transcendental." (B118) From this, Kant must deduce how pure concepts of the understanding are necessary to preserve objective validity for judgments while grounding it *a priori*. A science utilizing the pure concepts and grounded on intuition, such as Geometry, "nevertheless follows its secure course through strictly *a priori* cognitions without having to beg philosophy for any certification of the pure and lawful pedigree of its fundamental concept of space." (B120) When pressed to provide its entitlement to "the external world of the senses," Geometry answers quickly that "space is the pure form of its intuition, and in which therefore all geometrical cognition is immediately evident because it is grounded on intuition *a priori,* and the objects are given through the cognition itself *a priori* in intuition (as far as their form is concerned)." (B120) However, when these same questions of entitlement are applied to pure concepts of the understanding,

> [t]here first arises the unavoidable need to search for the transcendental deduction not only of them but also of space, for since they speak of objects not through predicates of intuition and sensibility but through those of pure *a priori* thinking, they relate to objects generally without any conditions of sensibility; and since they are not grounded in experience and cannot exhibit any object in *a priori* intuition on which to ground their synthesis prior to any experience, they not only arouse suspicion about the objective validity and limits of their use but also make the concept of space ambiguous by inclining us to use it beyond the conditions of sensible intuition, on which account a transcendental deduction of it was also needed above. (B121)

Because space and time are the only means by which an object can "appear to us...space and time are thus pure intuitions that contain *a priori* the conditions of the possibility of objects as appearances." (B122) Because of this necessary relationship between appearance and pure intuition, "the synthesis" in time and space "has objective validity." (B122) This unity of synthetic process will be elucidated later in this essay.

However, as stated above, compared to the pure intuitions of Time and Space, pure concepts of the understanding must give different reasoning to confirm their objective validity. For example,

> the categories of the understanding . . . do not represent to us the conditions under which objects are given in intuition at all, hence objects can indeed appear to us without necessarily having to be related to functions of the understanding, and therefore without the understanding containing their *a priori* conditions. (B122)

Space and Time, by the way they were given as part of the sensibility, are necessary *a priori*. Only by means of the Transcendental Aesthetic can a human be given intuitions of spaces and times, and thus interact with the world of experience. The categories, and thus the mathematical categories of Quantity and Quality, on the other hand, instead seemingly rely on "subjective conditions of thinking," when they should reveal objective validity and "yield conditions of the possibility of all cognition of objects" (B122). Kant thus argues that objects of sensible intuition must "accord with the conditions that the understanding requires for the synthetic unity of thinking" (B123) or in other words, the pure concepts of the understanding. This correlation between objects of sensible intuition and the conditions for synthetic unity of thought can only be justified through his process of transcendental deduction.

This deduction of all *a priori* concepts is important both for Kant's general argument and for his view of mathematics: "there are only two possible cases in which synthetic representation and its objects can come together...and...meet each other:

either if the object alone makes the representation possible, or if the representation alone makes the object possible" (B125). The case which Kant is most concerned with is the second one, where "the representation is still determinant of the object *a priori* if it is possible through it alone to cognize something as an object" (B125). The argument put forward by the Philosopher is that, since all appearances agree with the formal condition of sensibility, since that is the only way we can receive them, "the question now is whether *a priori* concepts do not also precede, as conditions under which alone something can be, if not intuitive, nevertheless thought as object in general" (B126). This case is correlated with the second condition for cognition, the concept, "through which an object is thought that corresponds to this intuition" (B125). To sketch the argument, Kant states that

> All experience contains in addition to the intuition of the senses, through which something is given, a concept of an object that is given in intuition, or appears; hence concepts of objects in general lie at the ground of all experimental cognition as *a priori* conditions; consequently the objective validity of the categories, as *a priori* conditions rests on the fact that through them alone is experience possible. (B126)

Experiential validity can only be confirmed through the use of the categories as *a priori* conditions of cognition. Thus, the understanding itself utilizes these concepts to make itself the foundation of experience, and thus to apply the categories to objects of experience, namely appearances. If this were not so, then several issues from previous writers reemerge in the "Critique." Kant gives Hume as the ur-example of falling into this error, as "the empirical derivation, however, to which both of them [Locke and Hume] resorted, cannot be reconciled with the reality of the scientific cognition *a priori* that we possess, that namely of pure mathematics and general natural science" (B128). To avoid this error, and thus to preserve both experience and scientific cognition, Kant roots cognition within the understanding itself, and thus in concepts and the categories. Only by understanding the deduction will the necessary correspondence between *a priori* mathematics and objects of experience be demonstrated.

Following the groundwork laid in the first section of the Deduction, Kant begins building up towards one of his argument's capstones: the synthetic unity of apperception.

> The manifold of representations can be given in an intuition that is merely sensible...the combination of a manifold in general can never come to us through the senses, and therefore cannot already be contained in the pure form of sensible intuition; for it is an act of the spontaneity of the power of representation, and, since one must call the latter understanding...all combination...is an action of the understanding, which we would designate with the general title synthesis. (B130)

Synthesis was before defined as "the action of putting different representations together with each other and comprehending their manifoldness in one cognition" (B103). Following up from that, Kant introduces the representation of combination, which "is the only one that is not given through objects but can be executed only by the subject itself" (B131). This peculiar property is what makes combination important to the author's argument:

> Combination is the representation of the synthetic unity of the manifold. The representation of this unity cannot, therefore, arise from the combination; rather, by being added to the representation of the manifold, it first makes the concept of combination possible. This unity, which precedes all concepts of combination *a priori*, is not the former category of unity; for all categories are grounded on logical functions in judgments, but in these combination, thus the unity of given concepts, is already thought. The category therefore already presupposes combination. We must therefore seek this unity (as qualitative) someplace higher, namely in that which itself contains the grounds of the unity of differing concepts in

> judgments, and hence of the possibility of the understanding, even in its logical use. (B131)

After setting forth the goal, Kant explains the original-synthetic unity of apperception by showing what unites all representations: "***I think*** must be able to accompany all my representations" (B132). Prior to all cognition, the representation given to us is that of intuition.

> Thus all manifold of intuition has a necessary relation to the ***I think*** in the same subject in which this manifold is to be encountered...I call it the pure apperception…or also the original apperception. [Pure apperception] produces the representation ***I think***, which must be able to accompany all others and which in all consciousness is one and the same, cannot be accompanied by any further representation. (B132)

Because

> the identity of the apperception of the manifold given in intuition contains a synthesis of the representations, and is possible only through the consciousness of this synthesis...the relation therefore does not yet come about by my accompanying each representation with consciousness, but rather adding one representation to the other and being conscious of their synthesis. (B133)

Invoking the "***I think***" gained earlier in the argument, Kant states that

> it is only because I can combine a manifold of given representations in one consciousness that it is possible for me to represent the identity of the consciousness in these representations itself...The thought that these representations given in intuition all together belong to me means, accordingly, the same as that I unite them in a self consciousness, or at least can unite them therein. (B134)

By utilizing the representation of combination in concert with "***I think***," Kant arrives at his first significant conclusion in the Deduction: "Synthetic unity of the manifold of intuitions, as given *a priori*, is thus the ground of the identity of apperception itself, which precedes *a priori* all my determinate thinking" (B135).

This principle “of necessary unity of apperception” (B135) leads to a necessary synthesis of the manifold in an intuition, “for through the I, as a simple representation, nothing manifold is given; it can only be given in the intuition, which is distinct from it, and thought through combination in a consciousness” (B135). Only by uniting intuition and understanding under the “***I think***” can one become “conscious *a priori* of their [terms of judgement] necessary synthesis, which is called the original synthetic unity of apperception, under which all representations given to me stand, but under which they must also be brought by means of a synthesis” (B136). Only the synthesis of intuition of the manifold and the concepts of the understanding under the unity “***I think”*** allows for determinate objects, such as a geometrical line, to exist as representations to me. mathematics thus must have a certain connection to real and existing objects as a pure concept of the understanding, since the synthetic unity was grounded *a priori*.

After demonstrating the necessity of synthetic unity of apperception for determinate thinking, Kant moves to show that this principle is what determines all uses of the understanding, as objects can only exist for the understanding with the manifold of a given intuition as a basis.

> The supreme principle of the possibility of all intuition in relation to sensibility was...that all the manifold of sensibility stand under the formal conditions of space and time. The supreme principle of all intuition in relation to the understanding is that all the manifold of intuition stand under conditions of the original synthetic unity of apperception. All the manifold representations of intuition stand under the first principle insofar as they are given to us, and under the second insofar as they must be capable of being combined in one consciousness; for without that nothing could be thought or cognized through them, since the given representation would not have in common the act of apperception, ***I think***, and thereby would not be grasped together in a self-consciousness. (B137)

An object, as defined by Kant, only emerges in cognition as“that in the concept of which the manifold of a given intuition is united” (B137). This synthesis under “***I think***” that brings together the manifold into an object relies on the unity of consciousness, for without this there could be no unification of representation.

> Consequently the unity of consciousness is that which alone constitutes the relation of representations to an object, thus their objective validity, and consequently is that which makes them into cognitions and on which even the possibility of the understanding rests. (B137)

Without the synthetic unity of apperception, our understanding falls apart due to its inability to combine disparate elements into a single, united object, and thus an object’s unity would not exist for the understanding at all. Through this synthetic process given *a priori*, the formal relation of representation to object is preserved as objective validity, since all consciousness and thus cognition requires the unity of apperception to function or to even be possible. Thus, it can be said that

> The first pure cognition of the understanding, therefore, on which the the whole of the rest of its use is grounded, and that is at the same time also entirely independent from all conditions of sensible intuition, is the principle of the original synthetic unity of apperception. (B138)

To make any judgement, such as cognition of objects in mathematics, requires this synthetic unity, since “the mere form of outer sensible intuition, space, is not yet cognition at all; it only gives the manifold of intuition *a priori* for a possible cognition” (B138). It is only through through a unification of the manifold that the object can possibly exist at all, as

> in order to cognize something in space, e.g. a line, I must draw it, and thus synthetically bring about a determinate combination of the given manifold, so that the unity of this action is at the same time the unity of consciousness (in the concept of a line) and thereby is an object (a determinate space) first cognized. The synthetic unity of consciousness is

> therefore an objective condition of all cognitions, not merely something I myself need in order to cognize an object but rather something under which every intuition must stand in order to become an object for me, since in any other way, and without this synthesis, the manifold would not be united in one consciousness. (B138)

The objective condition here is not any individual intuition from experience, but instead "the pure form of intuition in time" or "intuition in general, which contains a given manifold" and which "stands under the original unity of consciousness solely by means of the necessary relation of the manifold of intuition to the one ***I think***" (B140). This objective unity is called the transcendental unity of apperception, or the formal ability to unify apperceptions, and is distinguished from a subjective unity of consciousness by an *a priori*, or formal, objectivity. This necessity of connection, and thus the pure synthesis of the understanding, is "what grounds *a priori* the empirical synthesis. That unity alone is objectively valid; the empirical unity of apperception….has merely subjective validity" (B140). Not only is the *a priori* grounding necessary for empirical unity, but no determinate thought could come about without this synthetic unity of pure apperception. Only through this construction of objective unity of thought which grounds empirical synthesis does Kant avoid Hume's problems and thus his skepticism, since Kant is able to find an objective source of unity rather than needing to rely on custom or habit. Mathematics, if they want to lay claim to the title of an *a priori* judgment of the understanding, must necessarily partake in the pure synthesis of the understanding, and thus gain objective validity within the framework of intuitions and concepts. Furthermore, this can be the only solution to the problem of correspondence between mathematics and objects of experience, since it retains objectivity. To explicate this resolution, Kant must delve again into judgements, returning to the categories.

Judgements, as defined earlier by the author, were the mediate condition between the understanding and the intuition. Kant extends this definition of judgements through unity of apperception, so that when investigating

> the relation of given cognition in every judgement...as something belonging to the understanding...I find that a judgement is nothing other than the way to bring given cognitions to the objective unity of apperception." (B142)

Thus judgements are used primarily to distinguish "the objective unity of given representations from the subjective", (B142) or the "relation of the representations to the original apperception and its necessary unity" (B142). Every cognition, and thus every judgement based on cognition(s), must necessarily fall under the unity of ***I think***, "even if the judgement itself is empirical, hence contingent" (B142).

From this new information it can be asserted that "the manifold that is given in a sensible intuition" (B143) bases itself on the original synthetic unity of apperception, since "through this alone is the unity of the intuition possible" (B143). Pushing beyond this point, Kant links the manifold to apperception, as

> That action of the understanding...through which the manifold of given representations (whether they be intuitions or concepts) is brought under an apperception in general, is the logical function of judgments. Therefore all manifold, insofar as it is given in one empirical intuition, is determined in regard to one of the logical functions for judgment, by means of which, namely, it is brought to a consciousness in general. But now the categories are nothing other than these very functions of judging, insofar as the manifold of a given intuition is determined with regard to them. Thus the manifold in a given intuition also necessarily stands under categories. (B143)

This deduction of the necessary link between apperception, the manifold, and the categories will form the firm base of Kant's argument for correspondence. No judgment can be made that does not fit into the categories; all judgments bring given cognitions to the objective unity of apperception. With all his pieces in place, Kant must only justify his application of the categories to objects of experience, and show how this connection is necessary due to it allowing for the possibility of empirical cognition.

Summation

After exhausting the field of pure understanding through the Categories, Kant's question of necessary correspondence has been answered through the Transcendental Deduction. Under the Kantian framework, mathematics required three major deductions:

1. "Space and time are thus pure intuitions that contain *a priori* the conditions of the possibility of objects as appearances" (B122);
2. Objects of sensible intuition must "accord with the conditions that the understanding requires for the synthetic unity of thinking" (B123) which can only be the pure concepts of the understanding;
3. "Synthetic unity of the manifold of intuitions, as given *a priori*, is thus the ground of the identity of apperception itself, which precedes *a priori* all my determinate thinking" (B135); this builds directly to the "objective unity . . . called the transcendental unity of apperception, or the formal ability to unify apperceptions."

Only through the combination of all these deductions has mathematics gained an objectivity, since the Transcendental Unity of Apperception is "what grounds *a priori* the empirical synthesis. That unity alone is objectively valid; the empirical unity of apperception . . . has merely subjective validity" (B140). It is impossible for mathematics to not be objectively valid under Kant's framework; mathematics, as a mode of judgment, necessarily mediates between Intuitions and Concepts, and there is no other way to make calculations of Quantity or Quality. To go outside the Categories would be impossible for a human being, and thus there is no other way to relate Concepts of ideal mathematical objects to Intuitions of empirical objects except by means of Mathematical judgment. This also ensures mutual understanding of mathematical concepts; everyone has an objective understanding of the properties of triangles through the concept Triangle. Within the Kantian framework, correspondence is necessary, since an object's mathematical properties are determined only by cognizing it through the Categories.

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Ode to (Well) Spent Time

It's done, it's done
My time is run
What will I do next?
Hit law school and try my best

I'm glad I met you, and I'm glad you're here
Thanks for helping me escape my nadir
Into life, we must all be going
Not a single one of us are going to be throwing
Yet I can't picture this time without you all here
It seems so contingent, yet the necessity clear
So thank you for being dear
And I hope we re-meet when Time brings us closer

Dedicated to
Jones, Schultz, Perez, Holden
Cortright, Martinez, Smith, Riley, Doval,
Friendman-Biglin, Hamm, Tsukahara, Zepeda, Braun
Gardner, Cooper, Balma, Ahrens, Choate,
Campagnia, Torrecillas, Tate, Tomelloso, Braun