

Humor and Education: A Practical Model for Didactic Efficacy

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A Senior Essay submitted in partial fulfillment of the requirements for the degree
of Bachelor of Arts in the Integral Curriculum of Liberal Arts.

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April 9th, 2019

Appendix 0—“That Which Can Be Removed Without Harm To The Patient”

The thesis of this paper was quadripartite. Assuming the definition of “Liberal Arts” to be “speculative knowledge, which is studied for its own sake” that allows for “the capacity to create using reason” by “liberating—because they free the mind from error and aid it in its pursuit of truth” (Ladd, 23), this paper sought to enumerate other pertinent senses¹ of the term; specifically: 1) who should be taught the liberal arts? 2) what should their contents be? and 3) how should this task be accomplished? In addition to these expositional treatments of the senses belonging to “Liberal Arts”, the thesis of this paper sought to demonstrate its argument with a proposed course of study that corresponds with the points made therein. In other words: this paper was written with the intent of identifying a valid model of a Liberal Arts education, and then producing said model from the identification. The four tasks of the paper could, then, be restated as follows:

1. Make an argument for who ought to be taught the Liberal Arts.
2. Make an argument for the content of the Liberal Arts.
3. Make an argument for how this task of a Liberal Arts education is accomplished.

and finally,

4. Construct a model in accordance with the arguments presented.

The thesis of this paper was thus always essentially partitioned in quadripartite manner, and how the paper was divided in presentation was reduced to a matter of methodology alone.

The paper began by presenting a demonstrative example of the necessity of its conclusion. That is to say, the paper began with a literary and philosophical exercise on the nature of *signs*, *information*, *allusion*, and *absurdity* that posits educational efforts grounded in *relative significance* disallow the ability for meaningful communication (i.e., communication free

¹ The term here, opposed to its appearance in the main body of the paper (i.e. the demonstration), is used loosely. Loose enough so that the *sense* of a *sign* “Liberal Arts” may include ‘who ought to be taught it’.

from error²). The argument was, then, constructed in such a fashion that forces any rational criticism³ of the argument to concede the quadripartite thesis's metaconceit: that a Great Books, Liberal Arts education⁴ is necessary for the apprehension of meaning of *signs*. Stated differently: the thesis sought to demonstrate the necessity of a Great Books regimen by constructing an argument for the content of all education that can only be rationally refuted with the knowledge gained from a Great Books education. The paper delivered a refutation of pluralistic relativism as contenders for the Liberal Arts by denying the refuted their grounds of argument: that pluralistic relativism—the interpretation of *signs* disconnected from intended meaning—can lead to knowledge.

The demonstration presented in the paper proper, alone, leaves the quadripartite thesis unaccomplished. At best this paper has completed the first and second task of the program demanded of it: an argument for who ought to be taught the Liberal Arts, and an argument for the content of a Liberal Arts education has been presented above; however, no argument for how the aims of a Liberal Arts education should be accomplished, nor a model representing that argument, have yet been presented. These appendices will now work to complete the program of tasks set out by the quadripartite thesis of the paper.

An argument for how to accomplish the goals of a Liberal Arts education will be presented. First, *signs*, *sense*, and *information* will again be examined—but in this instance, in relation to *truth*, *error*, and knowledge, so that the aim of a Liberal Arts education—to free the mind from “error”—can be understood. Frege's definitions of *sign* and *sense* from *Sense and*

² All communication by an Author presupposes some sense to be communicated by the Author's work. Authors seek to reach their readers via their work, i.e., their communications. When anyone who “professes” about the Author's work, the “professor” becomes a kind of translator between the Author and the audience. The text then opens itself up to “translation errors” as the text becomes one level removed from the audience and the voice of the Author. Much of this can be intimated from the demonstrative part of the thesis, the following Appendix 1 will deal with this issue directly.

³ That is, ‘criticism that has relevant grounds for refuting the subject of its polemic’.

⁴ An education, as proposed in the demonstrative part of this thesis, consisting of primary source texts.

Reference will be utilized, as will Kant's definitions of *truth*, *data*, and *information* found in the *Critique of Pure Reason*. Next, different aspects of *truth* will be further examined so as to produce some kind of common grounds for *truth* across different traditions: namely, aspects of *truth* as a phenomenon, and aspects of *truth* as a correct correspondence between judgement and object. In doing so, this appendix will establish a criterion for 'error'. Finally, these appendices will apply its new criterion for 'error' (i.e., in relation to the established definitions), learned by exploring aspects of *truth*, to the allegory of the cave found in Plato's *Republic*, so as to complete an argument for a particular methodology in avoiding 'error'. This argument will consist of the whole of the argument for a seminar-based method, and accomplish the third task of the paper's program.

With the audience, content, and method of an effective Liberal Arts education argued for, a model will be constructed in accordance with the conclusions presented in the paper's demonstration and appendices. This model will outline a program of study, or a method, that corresponds to a contemporary iteration of of the paper's conclusions and should not be interpreted as a kind of 'canon'. It will be easily demonstrated in *Appendix 1* why the establishment of a canon of Liberal Arts education is impossible, as the extent of its content is predicated on the society the Liberal Arts education is at work on. While that statement is not intended to say, as one may infer from it, that "the nature of 'error', and thus correctness or phenomenological understanding, is necessarily different in each society", it is intended to say that "each society will have a different iteration of a Liberal Arts education that belong to it". This will be easily inferable from the nature of *truth* soon to be examined; let it rest here that what is under discussion is the iteration of Liberal Arts education immersed in the society of which it is written: *Western Liberal Arts*.

Appendix 1—Determinability as Information Limits and Integrals of Truth

An argument will now be presented for the correct mode of presentation of Liberal Arts education. In the demonstration of the paper above, questions of *truth* and *error* as they related to *signs*, *meaning*, and *information* were purposefully ignored. The reason for this omission was due to the nature of *information*, utilized in both sets of concepts, revealing different things as we examined its relation to the later set (*signs* and *meaning*) versus the former set (*truth* and *error*). A result of these consideration of *information* and *truth* will demonstrate that a lecture-based mode of presentation of the Great Books fails in its attempt at educating liberally, and has no way of communicating the meaning of words without *error*.

While examined in relation to *signs* and *meaning* in the demonstrative example of the paper above, *information* is transmitted by the communication of *signs*. As noted above, the objectivity of *information* is the operative element that allows *allusions* to have *meaning*. Recall that Immanuel Kant defines *information* or *data*⁵, as 'mental content' in the *Critique of Pure Reason*, noting

“The person blind from birth cannot form the least representation of darkness, because he has no relationship with light; [...] All concepts of negations are thus derivative, and the realities contain the data, the material, so to speak, or the transcendental content, for the possibility and the thoroughing determination of all things.” [CPR, A575/B603]

And that *information* gained from *data* is empirically sourced but is itself not empirical in nature;

Kant's own footnote explicating the meaning of *data* in the quoted passage, he makes a clear association between *information* and *data*:

“The observations and calculations of astronomers have taught us much that is worthy of admiration, but most important, probably, is that they have exposed for us the abyss of our ignorance, which without this information human reason could never have imagined' to be so great; reflection on this ignorance has to

⁵The two are generally used interchangeably, but it should be noted that Kant is intending *data* to be only an individual's transcendental content. With this reading, *data* would correspond to Frege's *conception* as an individual's mental activity of a particular *sense*: “one need have no scruples of speaking simply of *the sense*, whereas in the case of a conception one must precisely indicate to whom it belongs and at what time” [*Sense and Reference*, p. 30]. *Information* in this paper is *data* in its impersonal, universally communicable form.

produce a great alteration in the determination of the final aims! of the use of our reason.” [CPR, A575/ B 603]

Information is what the human mind works with when construing thoughts with our faculties, such as Reason or Creativity. As the “transcendental content”, *information* would also be what forms the *sense* or ‘meaning’ of a *sign* to Frege⁶, as discussed in the footnotes of the demonstrative portion of the paper. *Information-as-sense* is thus how an objective meaning is communicated. This is all discussed in the footnotes of the demonstrative portion, and rehashed here for prosperity. The same relation does not hold when discussing *information* in respect to *truth* or *error*.

It is not immediately clear what role *information* has in relation to *truth* and *error*; moreover, it is not immediately clear what a definition of *truth* here can be. Certainly Immanuel Kant’s definition of *truth* is not instructive of how *information* relates to it. Kant takes the nominal definition of *truth* as, “the agreement of cognition with its object,” (Kant, *CPR*, A58) but soon finds issue with the statement when applied widely. He argues that for the nominal definition of *truth* to hold consistent universally invokes a paradox:

If truth consists in the agreement of a cognition with its object, then this object must thereby be distinguished from others; for a cognition is false if it does not agree with the object to which it is related even if it contains something that could well be valid of other objects. Now a general criterion of truth would be that which was valid of all cognitions without any distinction among their objects. But it is clear that since with such a criterion one abstracts from all content of cognition (relation to its object), yet truth concerns precisely this content, it would be completely impossible and absurd to ask for a mark of the truth of this content of cognition, and thus it is clear that a sufficient and yet at the same time general sign of truth cannot possibly be provided. Since above we have called the content of a cognition its matter, one must therefore say that no general sign of the truth of the matter of cognition can be demanded, because it is self-contradictory. (Kant, *CPR*, B83)

A general and certain standard for the *truth* of all thoughts is impossible to Kant: the very fact that it is a universally applicable standard of asking for the agreement of *any thought* to *any*

⁶ C.F. Footnote 20 in the demonstrative portion of the paper.

object asks that the content of the thought—the essence of the judgement—be disregarded. In short, Kant points out that asking for a “general criterion of truth” is paradoxical because asking for a “general” criterion of truth forces one to disregard what one would use as that criterion, the relevant *information*. Instead, Kant claims that a universal criterion of *truth* is only something that applies to the human understanding and logic: a non-paradoxical general standard of *truth* to Kant is a matter of “the agreement of a cognition with the general and formal laws of understanding and reason,” (Kant, CPR, A60). A universally consistent standard of *truth* holds not of *information*, but of *logic*, or the form of how *information* is handled. A statement that would fail the general criterion of *truth* to Kant would not be an assessment of *information* or content of a thought at all, but only how that content is utilized in a predicate in respect to its definition. An example of such a failure would be something akin to the statement of “the woman was a married bachelor”. In the statement, no judgement about the *objective validity* of the *information* can be made outside of the form; meaning, no assessment of the *truth* of the titles ‘woman’, ‘married’, or ‘bachelor’ can be made. Instead, what may reasonably be declared *true* or *false* is relation of the subject of the sentence to its predicate: ‘married’, ‘woman’, and ‘bachelor’ cannot fit the arrangement, or “form”, they are placed and still be in accordance with their definitions. Importantly, the thought-contents themselves are not up for judgement, but merely if their arrangement logically follows. It is in this way one can reasonably be said to be in *error*: “the error that concerns not form but content cannot be discovered by any touchstone of logic” (Kant, CPR, A60). Logical *errors* can be declared *true* or *false* (as the logic itself has no *information*, merely utilizes it), but *errors of information*, for Kant, are impossible.

However, while *information* and *data* were greeted without complication before, they now carry with them a question of usage when deriving *truth* from *error* as a Liberal Arts education aims to do. Namely: if a general criterion of *truth* is impossible for *information* (the

transcendental content of thought), and *information* is what is communicated by *signs*, does that mean no *error* of signification can occur in respect to the meaning of a *sign*? This is an issue if a Liberal Arts education seeks to “free the mind from error and aid it in its pursuit of truth” as *truth* appears merely a matter of logical validity irrespective of content validation. Assuming we take for granted the demonstrative portion of this paper to be persuasive—that if meaning matters, the reader is conceivably convinced that the content of a Liberal Arts education needs to be a Great Books regime—how can a Liberal Arts education accomplish its task? Above, this paper made the argument that this was accomplished by establishing the meaning of *signs* by tracing the genealogy of its *information* to the source of the *sign*, thus affording its user the non-mutated *information* it conveyed. This also allowed *allusions* to function in language, and was shown to be impossible without the ‘true-ness’ of *information* belonging to a *sign*. Kant’s rebuke of a general criterion of *truth* for *information* as it corresponds to an object appears at first as though it may apply to *signs* just as well. It may follow, then, that Kant’s rebuke of ‘*truth* of *information*’ would apply to this paper’s argument as well. Again, assuming the paper’s prior demonstration persuasive, what, then, becomes the *utility* of the Liberal Arts education towards freeing the mind from *error*?

Simply put, *information*, as the content of thought utilized by logic, is a necessary element for the discovery of *truth* or *error*. Without *information*, the criterion of *truth* would be fruitless and provide no knowledge. Kant cannot be taken to mean that, because there is no general criterion of *truth* for the agreement of any cognition with any object, that *truth* is worthless towards understanding objects. If Kant did mean that the logical criterion of *truth*—the agreement of cognition with the general laws and form of the understanding—disallowed any judgement of the objective truth of cognition, then one would never be able to declare *any* judgement as *correct* or in *error*. One could never make the assessment that “a married woman

cannot be a bachelor” because one would only have the empty form of judgement, there would only be a missing subject and an empty predication: “_____ is _____”. There would be no definition made possible of “woman”, nor of “bachelor”, and all *signs* would be equivalent in value. This is obvious, but must not be overlooked: just because there is no possibility of a general criterion of *truth* for objectively valid statements, does not preclude particular criterions of truth. Indeed, Kant is arguing against the opposite of this assertion:

since the mere form of cognition, however well it may agree with logical laws, is far from sufficing to constitute the material (objective) truth of the cognition, nobody can dare to judge of objects and to assert anything about them merely with logic without having drawn on antecedently well-founded information about them from outside of logic (Kant, CPR, B85)

Here, Kant is on no way disqualify the “(objective) truth of the cognition” when it is established on “antecedently well-founded information” about the object in question. *Information* is necessary for establishing *truth* and correcting *error*. This paper’s project of *signs* is not rendered moot, but saved, by Kant’s view of *truth*; what remains now is to explain how an *objective truth* of the *information* carried by *signs* might be described, so that the Liberal Arts education has some means to test its content.

Two common views of *truth* beyond what has been covered so far will help inform what remains of this task. As a convenient dichotomy, this paper will hold that *truth* can either be generally described as a matter of *phenomenology* or *correspondence*. Phenomenological *truth* describes the experience of a phenomenon; it is by any other name *truth* as genuineness or *truth* as illumination. *Truth* as correspondence is a one-for-one relation between thought and reality⁷. Importantly, neither view represented directly contradicts the Kantian notion of *truth*, but each view has significant implications on how one might handle epistemological questions of validity, and is why this paper now turns to them. These two different *senses* of the term ‘*truth*’

⁷ David Arndt, *Questions of Truth: A Genealogy of Liberal Education*, 2019

will set a standard for what an *objective truth* of the *information* carried by *signs* will have to pass to be able to be termed *truth*.

Phenomenologically, *truth* is a subjective process of revelation and becoming. Hegel describes such a *truth* in his *Phenomenology of Spirit* first as the “true shape in which truth exists can only be the scientific system of such truth” (Hegel, PoS, §5). This “scientific system of such truth” is a process of discovery where phenomena experienced by the human being leads to more and more refined versions of judgements and comprehensions:

To judge a thing that has substance and solid worth is quite easy, to comprehend it is much harder, and to blend judgement and comprehension in a definitive description is the hardest thing of all. (Hegel, PoS, §3)

In this “systematic” approach to *truth*, phenomena as “things” are assessed, considered, and then described based on prior experiences of in a integrated system of knowledge. However, what makes this *phenomenological* sense of *truth* initially irreconcilable with the *correspondent* sense of *truth* is its inherent subjectivity. While accruing *data* via sense-certainty in this systematic approach to *truth*, observed objects are converted to universal representations of themselves—so as to be *information* and the content of thought—and the object’s own objectivity is supplanted by the universal representation of the object in the “I”:

When we compare the relation in which knowing and the object first came on the scene, with the relation in which they now stand in this result, we find that it is reversed. The object, which was supposed to be the essential element in sense certainty, is now the unessential element; for the universal which the object has come to be is no longer what the object was supposed essentially to be for sense-certainty. On the contrary, the certainty is now to be found in the opposite element, viz. in knowing, which previously was the unessential element. Its truth is in the object as my object, or in its being mine [Meinen]., it is, because I know it. [...] We have now to see what experience shows us about its reality in the 'I'. (Hegel, PoS, §100)

Through the phenomenological experience of empirical *data* becoming *information-as-knowledge*, the nature of the *truth* in this system of thought is fluid. While before the object itself was the “essential element in sense certainty”, its importance to the “reality in

the 'I' becomes secondary as the “the universal which the object has come to be”. In this way, ‘pure’ objectivity is supplanted by subjective objectivity: objective reality itself is subsumed into the ‘I’ of the viewer, the subject, and *information* is necessarily divorced from objects external to the subject⁸. This subjective nature to the phenomenological view of *truth* is endorsed by Hegel explicitly:

The force of its truth thus lies now in the 'I', in the immediacy of my seeing, hearing, and so on; [...] I, this 'I', see the tree and assert that 'Here' is a tree; but another 'I' sees the house and maintains that 'Here' is not a tree but a house instead. Both truths have the same authentication, viz. the immediacy of seeing, and the certainty and assurance that both have about their knowing (Hegel, PoS, §101)

Each ‘I’, to Hegel, has an equally valid claim to their version of *truth* in that they both “have the same authentication” of their *information*. The to ‘I’s systematic understanding of their realities are both built from knowledge informed by phenomenological *data*, of their own experiences, and thus cannot rightly said to be false. Indeed, it is hard to imagine either ‘I’ being in *error* in their use of *information*, except that maybe the knowledge established by their experiences being out of an orderly arrangement. It is important to note that the phenomenological aspect of *truth* does not, in its subjective treatment of *truth*, invalidate the *objective truth* of *information*; instead, what the fluid nature of *truth* considered phenomenologically shows is that *truth* as experience is inherently subjective, i.e., there is an objectivity to the subjectivity of the *truth* of its *information*. However ‘true’ the *truth* of the phenomenologists’ experiences may be, though, their statements of *truth* will always lack an objective alignment between *information* and reality that the correspondence view of *truth* maintains.

A correspondent aspect of *truth* is an objective one-to-one relation of thought to reality.

What is meant by this is that *information* is in an equinumerous relation to an object, so that for

⁸ It may be reasonably objected that *information* is still intrinsically tied to the object that was initially recorded as *data*. This objection is not the view advanced by the above argument but, as it will make the approaching conclusion easier to sell to some, this paper sees no harm in letting the objection stand.

every expression of *information* carried by a *sign* that refers to an object, that object is expressed in the *information* carried by said *sign*. Frege, in his *Foundations of Arithmetic*, gives an example of such equinumerous correspondence between different objects:

If a waiter wishes to be certain of laying exactly as many knives on a table as plates, he has no need to count either of them; all he has to do is to lay immediately to the right of every plate a knife, taking care that every knife on a table lies immediately to the right of a plate. Plates and knives are correlated one to one (Frege, *Foundations of Arithmetic*, §70)

However, while Frege's example given above is a one-to-one correlation of objects in an intuitable spatial relationship, such correlation need not be couched in the intuition, and indeed is actually a matter of logical agreement:

Each individual pair of correlated objects stands to the relation-concept much as an individual object stands to the concept to which it falls—we might call them the subject of the relation-concept. (Frege, *Foundations of Arithmetic*, §70)

The “relation-concept” possessed by any two objects—or an object and “the concept under which it falls”—is the essence of correlation to Frege. Removing “knives” and “plates” from the example given above, what the waiter would be doing was correlating nothing to nothing—but a correlation still takes place, albeit an empty one. If asked, the waiter could correctly say of the empty table, “for each knife put down a put a plate”. The “relation-concept” of correlation holds even when void of content. Frege seems to agree that this “relation-concept” is the empty form of logic that is Kant's general criterion of *truth* above, as it is only the logical form:

The doctrine of relation-concepts is thus, like that of simple concepts, a part of pure logic. What is of concern to logic is not the special content

that is, the *information*,

of any particular relation, but only the logical form. (Frege, *Foundations of Arithmetic*, §70).

Frege's “relation-concept” of correlatability is extendable to the judgement of *information* with objects insofar that correlatability may be extended to “individual object stands to the concept to which it falls”.

This is the 'objective meaning' of a *sign* as *information* dealt with heavily in the footnotes of the demonstrative portion of the paper. Recall that each *sign* has a *sense*, per Frege:

The regular connection between a sign, its sense, and its referent is of such a kind that to the sign there corresponds a definitive sense and to that in turn a definite referent, while to a given referent (an object) there does not belong only a single sign (Frege, *Sense and Reference*, §27).

These *senses* of *signs* are independently referable objects themselves. Although there can be disagreement about the *sense* of a *sign* (C.F. Frege, *Sense and Reference*, §27 footnote 2), any one *sense* is not dependant for its content by an individual's *conception*, to Frege, the "internal image, arising from memories of sense impressions" (Frege, *Sense and Reference*, §29). The conclusion drawn in the footnotes of the demonstrative portion of the paper above is that a *conception* is composed of *data*, while this independently-referenceable *sense* (as it is still cognitive content) is *information*. This conclusion is maintained here, and from it can be easily demonstrated the validity of the correspondent aspect of *truth*.

With sufficient empirical *information* and the correct application of logic forms, objects may be correlated to *information* one-to-one as their relation-concept, just as objects "fall under" concepts. The object "Moby Dick" falls under the concept of "whale"; further, it can be said that there is "1" Moby Dick. In this, the object has been correlated to *information*: the actual reality of the object "Moby Dick" has been mapped to the cognitive content of "whale", and a number belonging to the cognitive content "whale" can be said of the object "Moby Dick", the number "1". As the waiter has correlated knives to plates so do we commonly correlate real objects to *information* and assert their objective validity every day. Not to violate Kant's dictum, this relation-concept cannot be rightly asserted of without the applicable empirical *information*, i.e., if the waiter did not know what the objects falling under the concept of "plate" were, he would not be able to correlate knives to them with certainty. If the reader does not *know*, i.e. possesses the *correct and true information* of what "Moby Dick" is, it will be difficult for them to answer how

many “Moby Dick”s there might be. This is the essence of the correspondent aspect of *truth*, the mapping of *information* one-to-one with reality.

It ought to be evident now why these two aspects of *truth* are pertinent towards the task of this appendix: each aspect of *truth*, perfectly valid, possesses conditions that appear to mutually exclude the other’s authentication. Phenomenological considerations of *truth* assert the essential subjectivity of knowledge and *error*. Correspondent considerations of *truth* assert the essential objectivity of knowledge and *error*. Both are true qua *truth*, however: the ‘I’ mediates phenomenological *data* into universal *information*, while objects may only ever really be called ‘known’ if the cognitive content a knower ‘possesses’ of them is found to agree with—correlate to—the object itself. The contradistinguished characteristics of these two aspects of truth do not merely represent a scholastic epistemological quandary, but a true didactic obstacle towards the transmission of any *sign*’s meaning.

A well-known example will aid in the demonstration of this didactic obstacle. In Plato’s ‘allegory of the cave’ found in Book VII of his *Republic*, Socrates and his interlocutor Glaucon discuss a metaphor for education based on their earlier discussions on the nature of knowledge: “‘Next,” said I, “compare our nature in respect of education and its lack to such an experience as this” (Plato, *Republic*, Book VII, 514a). Socrates proceeds to set forth an allegory where a person without education is like someone chained to the wall of cave, shown shadows of objects cast on the wall of the cave behind a partition, and told that the shadows of objects are truly the objects themselves (514a-c). Socrates says that these prisoners represent the human condition (515a), and that, the prisoners able to communicate but limited to their own view of the puppet-show, they come come to believe that “the truth is nothing other than the shadows of those artifacts” (515c). Socrates then describes the journey of someone released from their bonds and “cured of their ignorance” (515c) as someone drawn to the sources of light, all the

while seeing the objects he was denied before he'd be at a loss, as he had believed prior that those objects were "truer than the ones now being shown" (515d). Socrates then has "someone" compel the freed prisoner to "look at the light itself" (515e) and then drag the freed prisoner up out of the cave into the sunlight (516a), where, after a period of adjustment, the prisoner would be able to see the objects around him and the source of light, the sun (516b), where the prisoner would naturally "infer and conclude" on his own (516b). Socrates concludes that the prisoner's journey through the cave into the sunlight shows that "Education isn't what some people declare it to be, namely, putting knowledge into souls that lack it, like putting sight into blind eyes" but rather, "the power to learn is present in everyone's soul" and that "education is the craft concerned with this very thing, this turning around" (518c-d). The unspoken implication in Socrates's narrative throughout is that the "someone" who frees the prisoner and compels them to look into the light before dragging them out of the cave is the philosopher, and perhaps are those priorly freed prisoners who have grouped their way back into the darkness to free their fellow inmates.

Without undoing Plato's optimistic vision of education, the didactic problem identified earlier creates some real problems for the allegory of the cave. Allow us to assume the role of the philosopher and the liberator: we grope our way through the darkness and free our fellow man. Forcing his eyes into the light and dragging him up into the sun of the exterior he resists: "you are harming me", he utters. How can this man be speaking anything but the *truth*? His subjective experience confirms it, as does his correspondent deduction: for he is only under the criterion of avoiding *error* in relation-concept of subject of speech to predicate when he speaks to *truth*. He experiences pain by our hand, confirmed by whichever criterion of *truth* we use, and even if we profess some superior knowledge that might justify our actions, how do we transmit this with any kind of certainty that trumps the prisoner's pain? Worse still, what

grounds do we philosophers have to drag him into the sunlight? When *our* chains are somehow initially broken, how do we know with certainty that our groping path to sunlight will yield fruit? No test can exist that could ever hope to prove the *data* in someone's head matches the *information* actually correlatable to the objects external to us. Perhaps these issues could be overcome if there was a way to accomplish a synthesis of phenomenological aspects of *truth* with correspondent aspects of *truth*: an individual's *data* then could be compared to *information*. More remarkable, a conclusion could be reached contrary to the prisoner's assertion that we would be doing him 'harm' by causing him pain while teaching his eyes to see in the light. We could then connect objective reality to his subjective experience in a transgressive way. However, no attempt to make this argument without first admitting first this fact would be a lie of omission: no mere communication of *information* alone would accomplish this feat.

Plato acknowledges the impotency of the lecture-based model of education with his allegory directly. Education isn't "putting sight into blind eyes", but rather placing individuals in the circumstances to allow their eyes to see unhindered. Lecture could never do this: lecture would only ever be the philosopher returning to the darkness of the cave and ranting at his fellow inmates. Even if he convinced them the *truth* of this exterior world it would not change the circumstances of their imprisonment or the darkness before their eyes. Correspondent *truth*, while efficacious towards certain ends (notably, the scientific ends), still does not contain the utility of phenomenological *truth* within in, and so does not readily provide a response to the subjective argument it presents. The role of the philosopher, liberator, and educator can not be messianic in nature: it must be as *interlocutor*.

The allegory of the cave is more than a metaphor for the Platonic model of knowledge presented in the *Republic*, but also an argument for education by dialectic. The liberator drags the prisoners into the sunny exterior. The liberator compels the prisoner to look into the harsh

light. There is no 'lecturing' action to speak of here: the philosopher at all times is engaging their charge. Of course *information* is being transferred via *signs* and speech; no one would suggest that the axioms of Euclid could be taught efficiently by pantomiming and wrestling. But what is important in the allegory of the cave is that antagonistic involvement of the philosopher with their charge. Lecture does not engage the student in such a fashion, only dialectic will. Real education is an interactive, living process. It requires the use of more of our senses than is required by the lecture mode of presentation.

But now it may seem to the reader that this appendix is arguing for the preference of one version of *truth* over another: namely, of phenomenological *truth* over correspondent *truth*. This misunderstanding would be forgivable, as the language of *dialectic*—especially as the kind advanced by Hegel and opposed by Kant—does not immediately lend itself to the view of *truth* as a correspondence between *information* and objects as *objective validity*. This is not the case: neither is this appendix arguing for a preference of one aspect of *truth* in its *dialectical* model of education, nor does *dialectic* actually prefer phenomenological *truth* over correspondent *truth*. This must be shown.

The easiest path to the task at hand is to first show the possibility of synthesizing phenomenological *truth* and correspondent *truth*. This will accomplish several things simultaneously: first, it will show that the utility of the *dialectical* model of education is not only efficacious with regard to phenomenological *truths*, but, because the two *truths* have a *real correspondence*, also the correspondent *truths*; secondly, it will finally provide a full criterion for *error*, as whatever is neither phenomenologically *true* nor *correct* can in no way be considered *true*, and must be *false*; lastly, incidentally, it will provide a model for the *objectivity* of *information* carried by *signs* that this paper has so sorely lacked up to this point. Because *correctness* carries with it the stronger burden of proof (as phenomenologically *truth* only

requires that a knower's experience as *data* is universalized by the 'I' into *information*, regardless if the *information* corresponds to the object externally), this appendix will accomplish this task by showing the real possibility of a one-to-one, corresponding concept-relation between the two aspects of *truth*.

In some respects this appendix will be accomplishing very little in this regard: such a real correspondence between phenomenological *information* and objectively valid *information* was already demonstrated by Richard Dedekind. In his essay *Continuity and Irrational Numbers*, Dedekind sets out to establish a "scientific foundation for arithmetic" (Dedekind, pg. 1). Specifically, Dedekind was perturbed that the principles of infinitesimal analysis in calculus were all couched directly in geometric intuition:

[...] even the most rigorous expositions of the differential calculus do not base their proofs upon continuity but, with more or less consciousness of the fact, they either appeal to geometric notions or those suggested by geometry, or depend upon theorems which are never established in a purely arithmetic manner. (Dedekind, pg. 2)

Dedekind sought a system of arithmetic that was "scientifically" founded insofar that it was free from "geometric notions", i.e., intuitions, for its base. Presupposing the development of rational numbers and arithmetic, Dedekind went about defining a "system" of rational numbers R as a "well-arranged domain of one dimension extending to infinity on two opposite sides" (Dedekind, pg.5) with 'positioning' of the system R corresponding to distinct logical "laws", the most prominent being the partitioning of the system R :

If a is any definite number, then all numbers of the system R fall into two classes, A_1 and A_2 , each of which contains infinitely many individuals; the first class A_1 comprises all numbers a_1 that are $< a$, the second class A_2 comprises all numbers a_2 that are $> a$; the number a itself may be assigned at pleasure to the first or second class, being respectively the greatest number of the first class or the least of the second. In every case the separation of the system R into the two classes A_1, A_2 is such that every number of the first class A_1 is less than every number of the second class A_2 . (Dedekind, pg.6)

This separation of R into classes $A1, A2$ by number a allows Dedekind to construe system R , built with purely logical notions, to an identical definition of a geometric line:

If p is a definite point in L , then all points in L fall into two classes, $P1, P2$, each of which contains infinitely many individuals; the first class $P1$ contains all the points $p1$, that lie to the left of p , and the second class $P2$ contains all the points $p2$ that lie to the right of p ; the point p itself may be assigned at pleasure to the first or second class. In every case the separation of the straight line L into the two classes or portions $P1, P2$, is of such a character that every point of the first class $P1$ lies to the left of every point of the second class $P2$. (Dedekind, pg.7)

Because of the similarity of the two definitions—that is, the system R 's sameness to the line L , both in their essential features and how they are navigated—Dedekind can safely make the claim that the two defined objects transgress from analogy to a correspondence when an origin point is selected to represent 0 on line L as o :

With the aid of the latter to every rational number a a corresponding length can be constructed and if we lay this off upon the straight line to the right or left of o according as a is positive or negative, we obtain a definite end-point p , which may be regarded as the point corresponding to the number a ; to the rational number zero corresponds the point o . In this way to every rational number a , i. e., to every individual in R , corresponds one and only one point p , i. e., an individual in L . (Dedekind, pg.8)

With the ability to show a real correspondence between number value changes in the system R and positional distance on the line L , Dedekind has already demonstrated for this appendix the bulk of the task at hand. Dedekind's correspondence between a geometric object (an object perceived through the *intuition*, and thus, phenomenological in nature (any line may be said to "appear"—an experiential term—to be a certain length *via* the *intuition*); with a purely logical definition (cognitive content divorced from any individual's *data*, or, pure *information*) demonstrates a real correspondence not only with line and number, but of phenomenological *truth* (the intuitable experience of the line) with *truth* as *correctness* (truth as correspondence between *information* and object). Numbers, as externally referenceable objects themselves, always and everywhere possess an internal *correctness* that is their identity. Frege means as

much as *correctness* when he discusses numerical identity: “When we have thus acquired a means of arriving at a determinate number and of recognizing it again as the same, we can assign it a number word as a proper name” (Frege, *Foundations of Arithmetic*, §62). The number ‘2’ is equally the number ‘2’ on Earth counting marshmallows as it is the number ‘2’ on Mars counting dust clouds. Relating a phenomenological understanding to something as discrete as number is of the highest significance towards finding a synthesis of *truth* as phenomenon and *truth* as correctness, the next step being to find a general hypothesis where all statements of correctness may be reduced to a numerical format of some kind.

Fortunately, Frege puts forward a hypothesis kind to this endeavor when he suggests that the referent of all sentences is their *truth value*. In *Sense and Reference*, Frege hypothesizes that, since the driving cause of advancing from the *sense* of a sign to its *referent* is mankind’s “striving for truth” (Frege, *Sense and Reference*, §33), the real *referent* of any sentence—grammatically correct collection of *signs*—is its *truth value*:

We are therefore driven into accepting the *truth value* of a sentence as its referent. By the truth value of a sentence I understand the circumstances by which it is true or false. There are no further truth values. [...] Every declarative sentence concerned with the referents of its words is therefore to be regarded as a proper name, and its referent, if it exists, is either the true or the false. (Frege, *Sense and Reference*, §34)

Frege postulates that the heart of all declaration is simply a *truth value*, a simple ‘yes’ or ‘no’ to the question on whether the declaration has some corresponding correctness in reality. While normally a *sign* has an object as a *referent*—it is simply what a *sign* refers to (Frege, *Sense and Reference*, §27)—all sentences when considered as *signs* take on binary *truth values* to Frege. This is useful to the present task, as it allows all sentences to be expressed numerically.

Conjoining what has been covered of '*truth as correctness*' and its correspondence to intuitable phenomenon, a model may begin to be put forward that deconflicts⁹ the two mutually exclusive aspects of *truth* presented earlier. Building off of Frege's hypothesis, let us consider all declarative sentences as either *true* or *false* in their *correctness*. All declarative sentences may be regarded as either having a *truth value* of "1" or "0" corresponding to *true* or *false*, respectively. Declarative sentences that predicate any thing about the same subject may be grouped together as a collection of binary values. For example, "Moby Dick"- is -a whale; -a fictional character; -a jelly donut; -the killer of Ahab; may be read as: "Moby Dick" 1101. The experience of intuitable phenomenon would be convertible in the same fashion, as any experience that can be spoken of and mediated by the 'I' from *data* into *information* could be likewise converted under the condition that these experiences contain subject-predicate formats. In this manner, these two aspects of *truth* a deconflicted to the point that they can be expressed in the same manner; furthermore, nothing more is required of them. The burden of absolute *truth* or *falsity* was never taken on here, only that judgements of objective validity and of subjectivity could be reconcilable so that 1) it could be shown that a *dialectical* model of education did not preclude *truth as correctness*, as is sometimes alleged, and 2) a common criterion for error could be made possible. "Mary saw Jesus raise Lazarus from the dead" has a definitive and objectively valid *truth value*, expressible as a binary value, regardless of whether any one person *knows* that value. However, valid criticism of the method presented is present that could forestall the presented solution.

Several considerations yet addressed leave this hypothetical union of two seemingly opposed aspects of *truth* wanting. Of immediate concern to this argument is that, first, no criterion for *error* in this model, nor even a definition of *error* in this model, has yet been

⁹ A military term, used to describe the act of clearing the airspace used by artillery elements, seems useful here.

provided. Further, the second immediate concern is that, while an argument for the correspondability of objectively valid *information* with subjective experience has been presented, the model does not account for, among many things, the fluid nature of phenomenological *truth*.

Now, to be clear, this appendix has in some sense accomplished part the initial task set for it: in exploring *signs*, *sense*, and *information* in relation to *truth* and *error*, a model for a Liberal Arts education was identified, namely, the *dialectical*, or seminar, model. This is the model that most closely corresponds with the analysis carried out on Plato's allegory of the cave. Everything accomplished since then has been in the service of uniting two fundamental aspects of *truth* together to show that a dialectical model of education would not favor one aspect of *truth* over the other. More concisely, then, what remains of the task is to show now a kind of criterion for *error* for our dialectical model of education. If the fluid nature of experience is also accounted for soundly in our model, then, better still, but not necessary to the task at hand. Let a simple criterion be supposed: *error*, in the seminar model of Liberal Arts education, would be asserting any *truth value* of a subject-predicate relation-concept that it did not possess.

How ought we know what a subject-predicate relation-concept does or does not possess? Let us return to the project of mapping *truth values* to numerical data to imagine a useful analogy of what this model might look like. Now, this binary is really *data*—cognitive content supplied from empirical subjective experience—as long as any experience is declarative. However, as it stands, it is *only data*, i.e., there is nothing yet about the model that allows one to say, 'this is *information*, or *the general form of data*' yet. To say that, the model would have to represent individuals' *data* as some kind of compilation, and, somehow, a sense of abstraction. Taking creative inspiration from Dedekind, then, let us take the compiled *data* of all individual *conceptions* in a *dialectical* conversation and map them to a number-line. We

justify this step by saying the binary value of an individual's *data* will translate to some readable numerical value, and that value will be the length of the line demarcated and represent their aggregate *data*. Allow us to do so horizontally on a Cartesian plane, stacking *data* lines of different individuals on the same subject atop each other, lengthening the line. We can now roughly compare *data* of individuals—what they can say about a subject—in amounts. Vertically we will place time values representing when each binary value was asserted of the subject-predicate relation-concept. When, exactly, some truth-value is held by an individual, *data in time*, is now described as a point on a cartesian plane, and the aggregate *data* of an individual on a topic at any point of time is now described by an area. Coincidentally, in representing an individual's aggregate *data* as an area contained by time, we may better describe the process of interlocutors in a dialectic. For, over the course of a seminar, statements and hypotheses will be consistently tested, refined, or eliminated while new insights will occur to the participants. An individual's area *data* aggregate will be constantly in flux with the change of moment to moment in the conversation. The analogical model allows for this, as *data* is now represented in such a fashion so that it can be both continuously augmented in number (not representing new participants in the dialectic, but new *conceptions* gained), but continually diminished through *diressis*, hypothesis-testing, or other recursive investigations of terms common to dialectic. A figure will emerge from *data* contrived so: a figure roughly equivalent to Figure AacE from Lemma II and Lemma III of Isaac Newton's *Principia*, Book 1, on the motion of bodies.

The fourth corollary of Lemma III of Newton's Laws in the first book of his *Principia* lays out an interesting consequence of the lemma, that sets of rectilinear parallelograms (Newtonian diagram of forces) consistently augmented in number and diminished in area will form a curvilinear limit. It states: "and therefore these ultimate figures are not rectilinear, but the

curvilinear limits of rectilinear figures” (*Principia*, Bk 1, Sec 1, Lemma III, cor. 4). A ‘limit’ in Newton's *Principia* is a mathematical description of the phenomena whereby rectilinear figures (or forces and motions) can be described as a curvilinear lines (or shapes). Newton, setting out to describe the ‘true motions of bodies’, uses the limit as a particular conceptual ‘device’ to mathematically describe static states of objects ultimately changing into a different state (i.e. motion). In this way, Newton is able to describe bodies in motion with the limit—and the mental operations used to construe it.

Lemma III’s enunciation is a statement concerning the ultimate ratios of infinitely multiplying and then evanescent pieces of a figure established in Lemma II: “The same ultimate ratios are also ratios of equality when the widths AB, BC, CD, ... of the parallelograms are unequal and are all diminished indefinitely” (*Principia*, Bk 1, Sec 1, Lemma III). The “same ultimate ratios” are the pieces of Lemma II’s figure; moreover, they are parallelograms inscribed and circumscribed around an arc (itself immediately indeterminate as towards some classification of shape [i.e. whether it be a piece of a parabola, hyperbola, or what-have-you], other than one side of any figure AacE, in our case aE, being curvilinear), their sides and bases parallel to the two rectilinear sides of the figure aA and AE respectively, and each parallelogram’s base equal (in the case of Lemma II only). In Lemma II, it is shown that by constantly diminishing the bases of these parallelograms while constantly augmenting their number, one would have a situation where the inscribed figure (e.g. inscribed figure AKbLcMdD) would become equal with the circumscribed figure (e.g. AalbmcdnoE). This is merely because the difference of the inscribed and circumscribed figures is the sum of the rectangles between each (that is, between the top of a the inscribed figure and the top of the circumscribed figure), and that this difference’s sum would therefore have the equal base of the parallelograms and the combined altitude of the all the differences, that is, of the figure’s altitude (e.g. Aa). But, as

this rectangle is by supposition diminishing, and therefore becoming less than any given rectangle, while another is being opposed in its place (for they are infinitely augmenting in number), the figure reaches a state of equilibrium where the circumscribed figure becomes ultimately equivalent to the inscribed figure, and, as per Newton, “all the more, the intermediate curvilinear figure” (*Principia*, Bk 1, Sec 1, Lemma II). Lemma III shows this same result when the bases of the parallelograms are unequal. Its first corollary makes clear that ultimate sum of these figures is indeed the curvilinear line ‘between’ the two inscribed / circumscribed figures, and that this sum is indeed the limit as named in corollary 4.

Allow us, in our analogical tying together of *data* to the figure, to substitute Newton’s model of forces represented by the parallelograms of AacE with the *data* of our interlocutor’s *conceptions* and reveal what is left: a *limit*. This *limit* does not belong to any one interlocutor like that *data* does. This *limit* is formed from the maintainable extent of the *dialectic* itself, and belongs to no one: it is, in essence, a representation of abstracted *data*. The *limit*, in our analogy, is *information*. But more than just vague ‘information’: recall that the *data* mapped before was composed of the *truth values* of a particular *subject* in a subject-predicate relationship, and this *subject* will be revealed to be no more or less than a proper *sign*. This *informational limit* contains within it what will be called here, for brevity, the *integral of truth* of any subject. *Truth* has a pull like gravity, every subject-predicate relation-concept that falls outside of the *information limit*—that is, a value not contained in the evanescent *data sets*—is in terms of human knowledge, something than can only be termed in *error*.

This highly construed, analogical model is not proposed baselessly. Granted, criticism, both on the arbitrary nature of representing hypothesized cognitive-content as numerical *data* over time, and then deciding to map said *data* to a graph so as to apply Newton’s geometric limit to it, is warranted. A more precise argument will be required in the future to even approach

sight of the threshold of cognitive science with such a wild hypothesis; still, however, this model (as analogy) is not put forward without some material supporting it. Both Kant and Hegel describe “determinability” of a concept as a kind of limit. In chapter three section two of the “Transcendental Ideal”, Kant declares that:

Every **concept**, in regard to what is not contained in it, is indeterminate, and stands under the principle of **determinability**: that of **every two** contradictorily opposed predicates only one can apply to it, which rests on the principle of contradiction and hence is a merely logical principle, which abstracts from every content of cognition, and has in view nothing but the logical form of cognition. (Kant, CPR, A572/B600)

Kant’s idea of determinability, in this regard, is akin to his general criterion of *truth* applied to what is contained in a concept. In the model construed herein, a criterion of *truth* for any particular *data set* or *integral of truth* is readily supplied and supported by his language of determinability of predicates positively maintained of concepts. Kant’s test of determinability implies something like our *information limit* when considered independent and outside the cognition¹⁰. Meanwhile, Hegel’s language on the topic of determinability expressible as a *limit* is even more pointed: “the specific difference of a thing is rather its limit; it is where the thing stops, or it is what the thing is not” (Hegel, PoS, §3). It would stand to reason then that, if phenomenological aspects of *truth* and *truth* as correspondence are mergeable—and Dedekind’s project implies they are—then determinability as a limit of a concept or a criterion for *error* might still hold true, even if the details of our specific model do not.

To conclude this appendix, we must return to Plato’s cave, as promised, and apply what we have learned there. Again, we are the philosopher, the liberator, and the educator. When we free our former fellow inmate, we compel them to look into the light and we drag them out of the cave. How do we accomplish this? Evidently, not by lecturing, but by engaging the prisoner

¹⁰ It is of no minor note to notice that Kant’s formal definition of *data* and *information* is not only supplied in the same section as determinability, but also to support the same thesis c.f. Kant, CPR, A575/B603.

directly. Further, we now have several avenues of assurance that when we do so, we may arrive at *objective information* as the *limit* of individual *data* in this *dialectic*. What occurs in this *dialectic* is not ephemeral, but—somehow, if not specifically correspondent to the analogy above—is actually quantifiable. These composite *truth values* quantified together represent the *integral of truth* of any one subject, that is to say, of any one *sign*. But while this *integral* itself is subjective, the *limit* of *data* is not, as it belongs to no individual *data set*, and is independent of the interlocutors. This independent *limit* is the objectivity of *information* of a *sign*, divorced from individual *conception*. Every value that falls outside of the *limit* can be interpreted as *false* in regards to the *sign* in question. Our *dialectic* now has a standard for *error* in regards to *signs*. Lecturing, possessing only the individual lecturer's *data*, does not. As we descend into the cave to educate, we can now be assured that a *dialectical* method will allowing *diuresis* and hypothesis-testing—both phenomenological and scientific standards of *truth*—that lecturing can never accomplish.

To transit back entirely out of metaphor, an argument has been established for a particular model to a Liberal Arts education has been presented as it pertains to *signs*, *sense*, and *information* and their relations to *truth*, *error*, and knowledge. In the analogy of the *integral of truth* and *information as a limit*, it can be seen how a dialectic-based seminar method can achieve standards of *error* that lecturing, because of its mode of presentation, can never accomplish. When we engage in the seminar, we form *information* that is communicable, impersonal, but formed from the individual *datasets* each participant possess.¹¹ No further

¹¹ The reader will not be surprised to find that *information as a limit* may be descriptive beyond mere analogy. Contemporary methods of machine-learning applications lack a spark, a “spontaneity of cognition” (Kant, *CPR*, A50f/B74f), that man possess, making Artificial General Intelligences, at the time of this writing, impossible. While traditional approaches suggest that the limitations are currently hardware or algorithm based, those approaches do not take into account the phenomenological nature of the human mind. A real correspondence between experiential phenomenon and value-sets must be determined to if the Daedalic project of AGI's are to proceed. This analogical suggestion—currently purposefully vague, and limited to arguments of education for this paper—is an example of alternate forms machine-learning applications might be comprised of in the near future.

argument for the seminar model is necessary: if the Liberal Arts are to free us from *error*, the seminar is the only candidate for form, as the Great Books are the only candidate for content.

Appendix 2—Genealogy of Information

Part 1: “Western” Liberal Arts

We must revisit our earlier position: that we will be examine Great Books in a “Western” Liberal Arts education, and explain the arbitrary cultural preference imposed.

From our proposed analogical model of the “limit of *information*” and the “integral of *truth*” of objects—the metaphorical computing of sense-certainty *data* abstracted into independently reference, universalized *information*—we have seen how competing cognitive content may be understood to synthesize through a dialectic. Individual *data* sets are continually diminished through *diaereses* and hypothesis-testing, while at the same moment continually augmented through inference, investigation, and redefinition. The rectilinear extent of these individual sets of mapped binary *data* then outline a curvilinear limit, notably independent from the individual *data* sets.

While we stand on the strength of this analogy for a seminar based education—as Plato might stand on the lip of his cave—there is no immediate rationale to restrict the metaphorical sense of the model to one- or another-culture. In fact, the argument would run counter to such restrictions: the very notion of invoking *truth* in the form of a proposition of “universal mechanics”¹² implies a general applicability that would reject, by definition, cultural chauvinism. What does not stand counterfactual to the model, however, would be the subjective nature of those original *data* sets.

Recall that the *data* sets were construed from binary *truth values* of any predicate said of any subject. The linguistic, spatial, and temporal limitations to these data sets are immediately

¹² Newton's *Principia*, Author's Preface

apparent. In short, the only way to intellectually engage these *data* sets would be in the form of a natural language of proper names. All natural languages are contextually dependent. Cultural chauvinism is not baked into the cake in our analogy, but our model is culturally dependent for the *information* carried by *signs*. It is no mystery why Socrates or Lao Tzu relies on cultural common-places and specific historical instances to create their dialectical arguments—Socrates through his interlocutors, and Lao Tzu through his poetry, respectively. For anyone to pretend that a system of rational knowledge is not inured in the cultural and historical background of their time is a myopia that deserves no serious treatment; however, that does not preclude one from saying such a cultural-specific language cannot grant access to *true* knowledge. The correspondent aspect of *truth* reveals the very opposite: the ability to demonstrate scientific knowledge is a testament to the objective independence¹³ of *information*. This highlights another imperative of any analogy for *truth*: it must explain both *Truth's* independence from the human being and its relation to the human being as subjects. Because the *data* sets are, by their very nature, limited in scope, any didactic approach that seeks to integrate these *data* sets outside of the context for which they were produced would be participating in a kind of junk-science, and *error*. No rational person will subject the *Pali Canon* or the *Bhagavad Gita* to critique consisting of arguments from the Old Testament, any more so than they would critique those texts on the grounds of *Beowulf's* conceits; in short, there is no evidence that the texts from different cultures in the absence of direct or indirect references are in any form of “dialogue” with one-another. The *Tao* and *logos* have no bearing on each other at the time of their presentation, and to discuss what the *Tao* means to Socrates is speculative in the extreme, and not proper to discussing what *logos* means to Socrates.

Outside and removed from any historical or tradition-based argument, there is an

¹³ That is, ‘independence from human caprice’.

assertable rationale for Liberal Arts educations being context-critical and context-dependent. Anyone who tries to read Confucius's *Analects* without first reading Lao Tzu's *Tao Te Ching* runs just as much of a risk as anyone reading Ellison's *The Invisible Man* without reading Marx's *Das Kapital*; or worse, of reading the Lao Tzu's *Tao Te Ching* informed by Ellison's *The Invisible Man*; the *information* carried by the *signs* within the Great Works of any one culture are prerequisites towards understanding the *information*—*allusions*, direct references, and indirect inferences drawn from reflections of unrelated Great Works in a culture's lexicon—of succeeding Great Works. Liberal Arts educations must be predicated in the historical, social, economic, and linguistic contexts that provide them their *information*, presented by sources contemporary with the Great Works themselves. Even if it were not-so-great, Thucydides *History of the Peloponnesian War* would have value in relation to understanding Plato's corpus. To do otherwise—that is, discussing a work divorced from its original *context*—would be substituting constituent *data* values for the *information* it circumscribes.

What this means, then, is that while many Great Books do hail from non-Western cultures, any educator would be remiss to ignore the cultural *context* of the traditions they come from and those they approach. Whoever would claim that the Islam's *Quran*, Averroes's *Tahafut al-Tahafut*, and Rushdie's *Satanic Verses* do not constitute evidence of the universality of the 'Great Books' designation is just as much of a fool as the idiot who says Homer's *Iliad*, Plato's *Republic*, and Cervantes's *Don Quixote* are likewise not worth reading. It is simply that to understand the genealogy of a *sign's* meaning, it must be examined in the *context* from whence it came. In the Integral Program, as currently taught, we learn Greek to understand grammar and not Arabic or Sanskrit, not because the latter are non-essential to *truth*; instead, it is an acquiescence to the fact that to study the *signs* of humanity and their *truth* would take longer than four-years, and so a specific *genealogy* of signs must be selected.

This is also not to say that intercultural dialogue is impossible, ill-advised, or nonsensical; rather, the opposite, eventually, becomes necessary. Reading Nietzsche's *Genealogy of Morals* without an understanding of Buddhism defangs Nietzsche's argument about "ascetic ideals"¹⁴ just as much as not understanding Christianity does to its invective on arguments Judaism. Cross-cultural dialogues are essential, however careful we must be when not misappropriating the meaning of *signs* where it would mutate the meaning nonsensically. For this reason, this paper has advocated "Western" Liberal Arts education: it is not the starting place, but a starting place. Further, our model would suggest, on a metatextual level, that the *information* from non-Western cultural contexts must, in the order that they are encountered by the West, be introduced as Great Works in the Western "canon", as their contact alone has had monumental influence on the development of Western thought; which is no more to say than that, eventually, non-Western works undoubtedly become *information* in the Western "canon". Buddhism was cited *en totale* by Nietzsche, Averroes's and Avicenna's commentaries on Aristotle were cited directly by Aquinas's *Summa Theologica*, and, working in reverse from the Western frame of reference, Hegel and Marx were cited by Mao Zedong¹⁵, showing likewise movement West to East. Our model of Great Books Liberal Arts education then becomes a kind of *genealogy of information*, tracing the *significant information* of a culture. Inclusion of any one Great Book in the *corpus* of Western Liberal Arts (as a real Western "cannon" would run counter-productive to our proposed model of *truth*), then becomes a simple question of practicality.

Part 2: A Practical Model of a Western Liberal Arts Education

¹⁴ C.F. Friedrich Nietzsche, *Genealogy of Morals*, First and Third essays. Numerous references to Buddhism, as, to Nietzsche, a plight on the life-affirm *will to power* of mankind analogous to Christianity for the East.

¹⁵ *On Contradiction*, Mao Zedong, 1937

It is beyond the powers of this author to present a Great-Books-based, seminar-focused program from scratch. Fortunately, there is no need to: the Integral Program as constituted represents such a curriculum! However, this is far from the end of the discussion. The Integral Program, if it is to serve as a *genealogy of meaning* standing on the Great Books, is at present incomplete. To demand that the Program is responsible for covering all of Western Civilizations' intricacies or notions is absurd, and no serious thought of that nature is being suggested here. What is not absurd is this: the Integral Program sorely requires an update.

When moving from *orthos* to *praxis* or theoretical to the actual, certain appeals are made to the "state of things as they are", which are, granted, only "the understanding one has of the state of things", but this practical move is unavoidable nonetheless. The geometries described by Euclid are intellectible and beyond this realms' versions of perfection; yet, when we implement them in crafts we can still make a cornerstone with the solids he describes. Plans are always moving asymptotically closer to reality in each stage of their implementation. It does not become egregious to suggest that the Integral Program needs to radically reassess itself, and whether it is meeting the goals it describes in a changing world. It should not, then, come as an affront to suggest that the Integral Program - and then also Saint Mary's wider Collegiate Seminar Program as a whole - requires a fundamental update to what it presents.

The stated outcomes of each course in the Integral Program curriculum and the Collegiate Seminar Program will be surveyed in the shadow of the considerations of this paper. A core assumption we will make is that we are "genealogy maximizing", i.e., under practical time and resource constraints we will be maximizing each program's ability to allow its students to apprehend the meaning of *signs* and the elimination of *error*. For the Integral Program, this is of

course a more arduous task, as this applies not only to art, literature, theology, or philosophy¹⁶, but also to math, science, and music¹⁷. For the Collegiate Seminar Program, something of an ‘open question’ exists as to what the extent of that program should be.

The Collegiate Seminar Program is not, as the Integral Program is, solely responsible for its students’ apprehensions of the world of meanings around them. The enumerated learning outcomes of the Collegiate Seminar Program are instead roughly described as follows¹⁸:

1. Critical Thinking
2. Written and Oral Communication
3. Shared Inquiry

However, the same arguments that applied to the Integral Program apply equally to the Collegiate Program. Unless every attempt at communication or shared inquiry is to end the same - *relativistic* moving of goal-posts, intellectual agnosticism, appetitive arguments, and rampant skepticism—some common ground for the method of these three outcomes must be maintained. Inferable from our arguments earlier, it is not enough that the same text is read by all if the meaning of its words are understood by none. A *context* to the meanings is required, or else even my beloved Shakespeare’s *Sonnets* becomes little more than aesthetic gobbledygook. If a student reads the *Ones Who Walk Away from Omelas* and decides, like Calicles¹⁹ argues, that there is nothing wrong with the oppression of that one pitiful child and that it is in fact real justice, the advocate for *justice* in the Collegiate Seminar are neigh groundless to oppose him. Who but the abject moral *relativist*, that vile opportunist of Corcyra,

¹⁶ And thus, hopefully, this paper escapes accusations of being unceremoniously lumped into “Semiotics”. I would not despair at this epitaph, but I hope that the project here undertaken, while perhaps being grounded in that tradition in approach, goes beyond it.

¹⁷ For clarity, the Program’s constitution - as a dedicated and prescribed curriculum in modern analogues of the traditional *trivium* and *quadrivium*, is here maintained. The structures of tutorials, laboratories, and seminars, is likewise maintained.

¹⁸ Saint Mary’s College of California, *Collegiate Seminar Learning Outcomes*, retrieved: <https://www.stmarys-ca.edu/collegiate-seminar/learning-outcomes>, April 1, 2019

¹⁹ Plato, *Gorgias*, 483b, 492a–c

would maintain that state of affairs is what Saint Mary's College of California - a *Lasallian institution* - aims to teach? No; to argue for this state of affairs is not intellectual humility, but intellectual chauvinism: it supplants *truth* for *entropy*. The Collegiate Seminar can not be held responsible for teaching its students as much as the Integral Program aims to, *but it must teach something more than catering solely to the relativist*. Some common language, some common *meaning* must be established before the Collegiate Seminar's outcomes may be met. A reading list ought to be construed that seeks to establish a truncated *genealogy* for the Collegiate Seminar.

The Integral Program itself, again, will not be reengineered outside of a few key considerations illuminated by this paper as theory comes in contact with reality. They are as follows:

1. The Integral's curriculum must supply its students with an understanding of the world around them as part of its program to free its students from *error*. However, it currently teaches nothing of post-Aristotelian formal logic, statistical analysis, operational analysis, or computer science. The bedrock of feminist and gender theory is absent from the Program, as are the foundations of sociology²⁰ Assuming time constraints alone are not the issue at hand, the Integral Program is stagnating in its mission for a seminar-based Great-Books Liberal Arts education. A few potential solutions for a handful of these deficits will be suggested here.²¹

²⁰ Outside of Marx's *Das Kapital*, whose ties to the former are debatable at best.

²¹ A fruitful solution to several of these topics that are, because of their extensive natures, not afforded room in the Program currently can be found in the idea of either *Integral Electives* or *Special Studies*. Presuming funding is secured to go beyond the current course limit, an elegant solution would be to either to have "Integral-style" electives for, say, computer science (tracing its genesis from Turing to modern applications, based on source texts and practicals), or even Special Study tracks that span multiple years (classical languages or mathematical logic as a sequence lying parallel to the Program's regular courses, for example). This idea is not further expanded on in this paper, as it advances on the territory of formal course proposals this paper is not prepared to make.

2. It is currently unclear if several extent areas of the program are meeting their designated course outcomes. Students report across all cohorts that cohort members are displaying insufficient recall of grammar rules, historical understanding, and intra-program mathematical knowledge. Maintaining the Program's prohibition against testing students, modifications to the training and development of tutors must be enacted to counter this.
3. The Integral Program has always been casted as a "Western" Liberal Arts education, a position defended by Appendix 2 Part 1 of this paper. However, this position must be re-examined insofar that the reach of the information age has blurred the lines of "Western Civilization". Aquinas refutes Averroes, and Nietzsche reimagines Zoroastrianism: without abandoning the earlier position taken in Appendix 2 Part 1, the reading list must be evaluated to see if integration of non-Western Great Books into the reading list can better be achieved.

Again, it is beyond the powers (or rights) of this author to attempt a wholesale revision of the Program's reading list or extent. From Integral Program's curriculum, a truncated list should selected as the Collegiate Seminar's reading list, loosely divided into the same class schema currently maintained by that program. Opposed to the currently reading list maintained by the Collegiate Seminar, this would allow a kind of dialogue between the two Programs while also protecting the Collegiate Seminar from its problems of listlessness and nihilism that it currently suffers from.

1. *Mathematics Tutorial*

In the Integral program, the Maths Tutorial sequence is intended to give the

student “insight into the fundamental nature and purpose of mathematics.”²² In a wider context, the Maths instructs the student in the foundations of pure mathematics, astronomy, and physics. However, the degree of importance attached to the development of mathematical thought versus general critical thinking is inconsistent at times, largely stemming from how pure mathematics’ connections with astronomy and physics is handled. Beginning with a rigorous foundation in Euclid’s *Elements*, the program skips a beat and transitions to Ptolemy’s *Almagest* before returning to more rigorous studies with Apollonius’s *Conics*. This is a poor strategy.

Ptolemy’s *Almagest* is, simply, in the wrong tutorial to the detriment of the Program. The introduction of the *Almagest* in the first semester of the Sophomore year defeats the purpose of the mathematics tutorial twice over. First, it serves as an extension of the summer malaise that haunts American education at large, separating the incoming Sophomores from their mathematical foundations in Euclid even more. Time spent between the *Elements* and *Conics* serves to rob students of practice with the style of the former in transition to the later. The *Almagest*—to which this author owes his first publication credit in defense of its value, coincidentally—is an astonishing work of brilliant analysis and rudimentary science. While it surely utilizes math, to call the *Almagest* a treatise on “math” is to call economics “math” on the same grounds. It is a treatise of natural science and belongs in the tutorial dedicated to natural science, the Freshman Laboratory tutorial.

Placing sections of the *Almagest* in the second semester of Freshman Laboratory is advantageous for several reasons. First, curating selections from the work is crossing no Rubicon itself, as already the voluminous treatise is not read in its entirety. It is a

²² The Integral Program, “About Our Curriculum”, retrieved April 1, 2019:
(<https://www.stmarys-ca.edu/the-integral-program/what-do-integral-students-study>)

chance to critically examine how much of the text is remunerative, and rend unnecessary fat from the flesh of the volume. Second, it fits the aims of the Freshman Laboratory tutorial's astronomy section already. To ask the Sophomores to contemplate the spheres once more is, if anything, simply inopportune when there is only so much to read. Third, it allows application of what is being actively learned in conjunction with Euclid's *Elements*, and requires little math additionally that may be supplemented easily. Perhaps most importantly, though, is that it opens up a whole semester's worth of mathematics training, allowing the reading list for the tutorial to shift and expand. The importance of Ptolemy's *Almagest* is not liable to be lost, and at most what will be incurred is a more rigorous Freshman Laboratory tutorial for the Freshman, with some works compacted into the first semester to compensate. What is to be gained by the move is disproportionate: an additional three months of mathematics instruction and a much-needed reordering of the geometry sequence so skills are retained more robustly. If nothing else is listened to in the whole of this paper, this must be taken into serious consideration.

A minor word more on the Freshman Mathematics tutorial. Retention of the *Elements* is paramount to this tutorial's aims. Too often a math tutorial is mired in its progression by a portion of the tutorial not recalling basic Euclidean geometry. This cannot stand; it is anathema to both this paper's argument and to the spirit of the Integral Program as a whole. What good is reading the Great Books if even the most fundamental members of that appellation pass from memory as water through a sieve, or even, grains in an hourglass? Certain venerable tutors are known to have compiled a list of "know forever propositions" pulled from Euclid, Copernicus, and Newton that, after a student has demonstrated them, can then be expected of by the student to provide a

(rough) demonstration of their conceits in any future tutorial. It is imperative that this list of “know forevers” be implemented, and students be held accountable to them. The very soul of the Program is at stake if they are not.

Sophomore year, now half-empty, may be otherwise devoted to its usual pursuits of Algebra and the Cartesian coordinate system. Again, the Program’s own words: “The rest of the year is devoted to Apollonius’ presentation of the conic sections, followed by Descartes’ Geometry. By the end of sophomore year students must demonstrate proficiency in basic algebra”²³. This requirement, like the last, must be enforced rigorously. With the rest of the year open, the challenge of calculus may be attempted, early.

The problem of the Program’s near-abandonment of calculus is a perilous one. Like Ptolemy’s *Almagest*, Newton’s *Principia* is a monumental work of the intersection of math and astronomy, both in length and import. Unlike Ptolemy, Newton’s *Principia* generates novel mathematics still in use today with the *limit* and the *integral*. With a semester available to begin the *Principia* early, it is tempting to make the same arguments as with the *Almagest*: fit it where it can be fit, ect. The idea, though, of students taking on the challenge of the *Principia* and accomplishing it within three months is hard to swallow - even with a more rigorous standard for the earlier math sequence. If possible, and the *Principia* can be painlessly worked through by the end of Sophomore year thanks to the earlier revisions of the Program, then, more to celebrate! This is doubtful though; what is more likely is that we are now able to split Newton in half by the end of Sophomore year—an argument that has pitfalls associated with it. It will ultimately prove more useful to do so than not: the now-free second semester of junior

²³ The Integral Program, “About Our Curriculum”, retrieved April 1, 2019: (<https://www.stmarys-ca.edu/the-integral-program/what-do-integral-students-study>)

year allows a true focus on an historical introduction to calculus, not only Leibniz's *Nova Methodus pro Maximis et Minimis*, but also the genesis of operational analysis with Louis François Antoine Arbogast's 1800 *Du Calcul Des Derivations* (although perhaps a more well-known substitute may be supplied in Augustin-Louis Cauchy's 1829 *Cours d'Analyse*), allowing the Program to assemble a course-reader that, through source texts, enables the Program to teach a robust calculus portion in the Junior Math tutorial.

With such significant changes to the mathematics tutorial, it becomes unclear what else may be adjusted remuneratively. The normal course then would be played through for Senior year with Lobachevsky, Dedekind, Frege, Poincare, Einstein and Feynman. Despite its importance, *correlation and dependence in the statistical sciences* are found to have no room to fit for senior year. To do so would be to trade Lobachevsky, the most likely candidate for removal, for technical knowledge with no clear Great Book to be based off. The model then rests for for the mathematics tutorial, the major revisions being the transitioning of Ptolemy's *Almagest* to its more appropriate home in the laboratory tutorial, the insistence of a "know forever" requirement to be decided by the tutors, and the addition of a more robust calculus portion.

2. *Language*

The Language Tutorial, at present, is divided into four years that each cover a distinct aspect of language structure and purpose. Freshman year's focus on grammar and syntax, sophomore year's Socratic and Aristotelian logic, Junior year's rhetoric, and Senior language's focus on the dialectic. Additionally, each year is currently accompanied by an additional area of focus of what could be called 'content': Freshman year's Greek and the fundamentals of translations, Sophomore year's shoring up of the Geco philosophic *corpus*, Junior year's poetry, and Senior year's exploration of

semantics. Because of the difference in the kind of topics each year explores it would be inappropriate to discuss each year together as we did Mathematics. Also, aside from stated deficiencies in grammar knowledge—a technical instruction the reader will know by now I am unqualified to pontificate on—this paper has little to improve on the Program’s first and second years’ tutorials in language²⁴. This paper will then sketch its model for rhetoric and poetry, dialectic and semantics roughly, as to give all four foci the same treatment the paper gave the mathematics would be both too lengthy for this paper’s purposes, and stretch the limits of its author’s skill. General dictums will be enumerated.

a. Poetry

The Program’s current iteration of the Poetics portion of the Junior Language Tutorial is sufficient, or rather, it is *not insufficient in any way that may be offered remedy of*. Poetry, that many-headed monster, is a hydra whose vague determinants nearly foiled this paper’s entire conceit²⁵. Because the body of Great Poetry is so endlessly vast even in paltry English, this paper would never be well-enough equipped to suggest a reading list beyond comprehensiveness as the one the Program already offers. Countless poems may be suggested as “Great and necessary additions”, too many for the limited time the Program is offered²⁶.

²⁴ The only obvious changes ought to be scaling back the redundancy of the Program where inopportune. Might Averroes be moved from Seminar to Language tutorial over a second reading of *Phaedrus* (one of three), or Al-Gazali over a second reading of *Phaedo*? These changes, again, would lose little and gain much, as they would open a reading in Sophomore year’s crowded seminar, in which of course either Chrétien de Troyes or Sir Thomas Malory could fill with either’s interpretation of the Arthurian legend, an addition that would give context to the irony of Cervantes later.

²⁵ Albeit, without Poe’s *The Philosophy of Composition* that accompanied his *Raven*, this paper would never have existed.

²⁶ Some additions perhaps warranted: the *Bhagavad Gita* and selections from the Buddhist *Pali Canon*, as they are not only beautiful and moving even translated, but were introduced to the West

The only substantial change our model might make is to migrate Milton's *Paradise Lost* from the Junior Seminar to the Language Tutorial. This migration would allow space for two readings to be added to the Seminar list, in which the vacancy ought to be filled with Voltaire's *An Essay on Universal History, the Manners, and Spirit of Nations* towards the end of Junior Year, as this would work to correct the dearth of historical understanding cohorts are reported to possess.

b. Rhetoric

Rhetoric, by contrast, has much to be done about it; or rather, *much that could be done with it*. As promised, I will keep my remarks brief. This portion of the course could, if the tutors so chose, be engineered to cover rhetorical aspects of of sociology and add ethnically diverse authors to the currently curriculum (Ibn Khaldun's *Muqaddimah* and W.E.B. Du Bois's *Souls of Black Folk* both stand out as examples to accomplish this feat), or perhaps add Simone De Beauvoir's *the Second Sex* and cover the foundations of feminist theory (itself a kind of rhetoric as much as the *Diary of the Life of Frederick Douglass* is) while examining the rhetoric of her essential work. Either way, the advantages ought to be clear, here: the Program has a chance to serve two masters in revising the Rhetoric portion, both the College's larger political direction and the Program's own standards of education. An American Diversity credit acquired here would allow students more freedom in selecting electives from the College, and perhaps allow for more interesting electives offered by the Program itself to gain appeal.

contemporaneously approximate to the Seminar reading list of this period, and has influence on later authors.

c. Semantics

As with Poetry, this author is unable to advise on the selection of works for this aspect of the Language tutorial. Favorable and unfavorable biases mar this author's judgement here, and it would do better for this paper's aims to remain silent.

d. Dialectic

A significant change is to be suggested here: the first semester, as the one devoted primarily to dialectic inquiry, ought to receive back the bulk of Hegel's *Phenomenology of Spirit* from the Seminar. Space will be made, again, with the suggestion of removing repetitive readings of Plato, Aquinas, Aristotle, and other authors visited a second- or third-time. Every Book read in the Program deserves a second or third reading, but there is only so much time in four years to read so many books once. Notably, this would free seven Seminars open to additions. This alone is argument enough for such a significant change, and no more need be argued here for it.

3. *Laboratories & Music*

The subject of the laboratories tutorials will prove to be a contentious one, but one unsolvable by any model put forward here. The model's alterations to the Freshman laboratory have already been discussed, this paper stands by those arguments with no addendums. However, improvements on the laboratory tutorial are necessary, as the rapid pace of scientific understanding has made the Junior and Senior lab sequences lag behind the Great discoveries of the modern era. However, the issue here is one simply of time; the great breadth of the scientific development can only be more properly covered with more time. This model would suggest - as care has been taken to

eliminate the need of an outside American Diversity credit from the College from the model's changes to the Junior language program - adding an additional laboratory module to the second semester Sophomore year, covering interim developments of sciences that comprise the first half of Junior lab. This would allow Junior lab to begin with Lavoisier's *Elementary Treatise of Chemistry* instead of Galileo's *Two New Sciences*, or perhaps even end Sophomore lab with the former, allowing the Electromagnetism module used at St. John's to be implemented here as well (with modifications). Ultimately, any alteration to the Program's curriculum by this model is going to point out the need for more time, rather than alter the course of a reading sequence dramatically through substitution or rearrangement.

4. *The Seminars*

Much has already been said about alterations to the Seminar reading list. The potential relocation of Averroes, the suggested relocation of Milton, and the wholesale migration of Hegel into their corresponding language seminars sacrifice only repetitions of Great Books already read, and ask for no sacrifices for their sakes. Their replacements - Malory and Voltaire for Averroes and Milton - help fill some awkward gaps in the curriculum's chronology much-needing filling. Hegel's places remain to be spoken for, and it has yet to be said as to what other replacements this model suggests. Without wasting more ink:

Hegel's removal opens up seven seminars, enough for a large work of literature (*Moby Dick*, *Name of the Rose*, *Infinite Jest*, etc.) or a major philosophical work (*Being and Time*, *Of Grammatology*, *The Second Sex*, ect.). Many considerations are present in this, e.g., if *Moby Dick* were added, ought *Billy Budd* be removed? if *Being and Time* or *Of Grammatology* were added, they would fall late in the year - is it wise for such

heavy weights to fall at the end of Senior term? Further, there were dozens of Great Books written between *100 Years of Solitude* 1967 publication year and current date - if we select a novel, which one do we select and why? This paper's stance is to return to its conceit: the Integral Program, being a curriculum of Great Books, is a Liberal Arts education. Liberal Arts education 'free the mind from *error*' by imparting the meaning of signs on their students—which, correctly done, is achieved through the dialectic of a Seminar. The importance of the *signs* and their meanings takes precedence over works of fiction too numerous to rank, and so works of philosophical import will be favored in our model. Out of those mentioned, Simone De Beauvoir's *the Second Sex* remains essential. Heidegger's *Being and Time* will also be chosen for its enduring effect on continental philosophy; to make room, repetitive authors will be selected out, specifically Virginia Woolf's *Mrs. Dalloway* and Heidegger's *The Question Concerning Technology*. Likewise replaced will be Freud's *Introductory Lectures on Psychoanalysis* with his own *Civilization and its Discontents*, as the latter covers much the same ground as the selections the Program currently reads, with the addition of the author's later *id*, *ego*, and *superego* theory of the self, as well as many facets of psychology still in practice today. The last alteration this paper suggests directly to the curriculum is, as the practice of this maximization strategy, reduce the final Phaedrus reading to one seminar, and put as the penultimate reading Albert Camus's *the Myth of Sisyphus*.

This paper considers its curriculum suggestions for the Integral Program completed.

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